Strategic Asset Allocation and Risk Budgeting for Insurers under Solvency II

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Introduction

Overview:

1. Solvency II regulation for insurers
2. Standard formula for the solvency capital requirement (SCR) for market risk
3. Derive the optimal asset allocation: general lessons learnt
4. Apply for representative life insurance company: numerical examples

Solvency II

Regulatory framework for insurance companies in European Economic Association (28 EU countries + 3)

- In force since Jan-2016

Key features

- Market-based valuation of assets and liabilities
- Solvency capital requirement (SCR)

Solvency Capital Requirement

- Insurer needs to have sufficient own funds such that it can cover all potential losses over a 1-year horizon with 99.5% probability
  - Own funds $\geq -$(Value-at-Risk at 0.5%)

Models to estimate the SCR

1. Solvency II standard formula
2. Internal risk model
   - E.g., extreme value distributions, joined by copula

Standard Formula for SCR

1. Applies "0.5% worst case scenario" shocks to each asset & liability on the balance sheet
2. Then aggregates the losses with a square-root formula, with given correlation matrix

- All parameters prescribed in the technical documents of the Solvency II regulations

Relevance for Thailand

- Office of Insurance Commission (OIC)
  - Risk-Based Capital (RBC) framework for insurance companies, <u>in effect since 2013</u>
    - Capital requirement based on VaR with 95% confidence
  - Latest draft of new Risk Based Capital 2 (RBC 2) framework released in April 2016
    - At the testing stage
    - Confidence level may be increased to 97.5% or 99.5%
    - Similar to Solvency II
    - Market risk module similar to Solvency II standard formula

SCR for Market Risk

Market risk types

The Solvency II standard formula for market risk determines the capital requirement for six types of market risk:

I. Interest rate risk : $SCR_{Mkt,I}$

II. Equity risk : $SCR_{Mkt,II}$

III. Property risk : $SCR_{Mkt,III}$

IV. Credit spread risk : $SCR_{Mkt,IV}$

V. Currency risk : $SCR_{Mkt,V}$

VI. Concentration risk : $SCR_{Mkt,VI}$

SCR for Property Risk

III. Property risk: $SCR_{Mkt,III} = \Delta_{prop} A_{prop}$

with Δ_{prop}=25%, and A_{prop} the total property value

Example:

A_{prop} = 1,200 million euro

$SCR_{Mkt,III}$= 0.25 x 1,200 = 300 million euro

SCR for Equity Risk

II. Equity risk

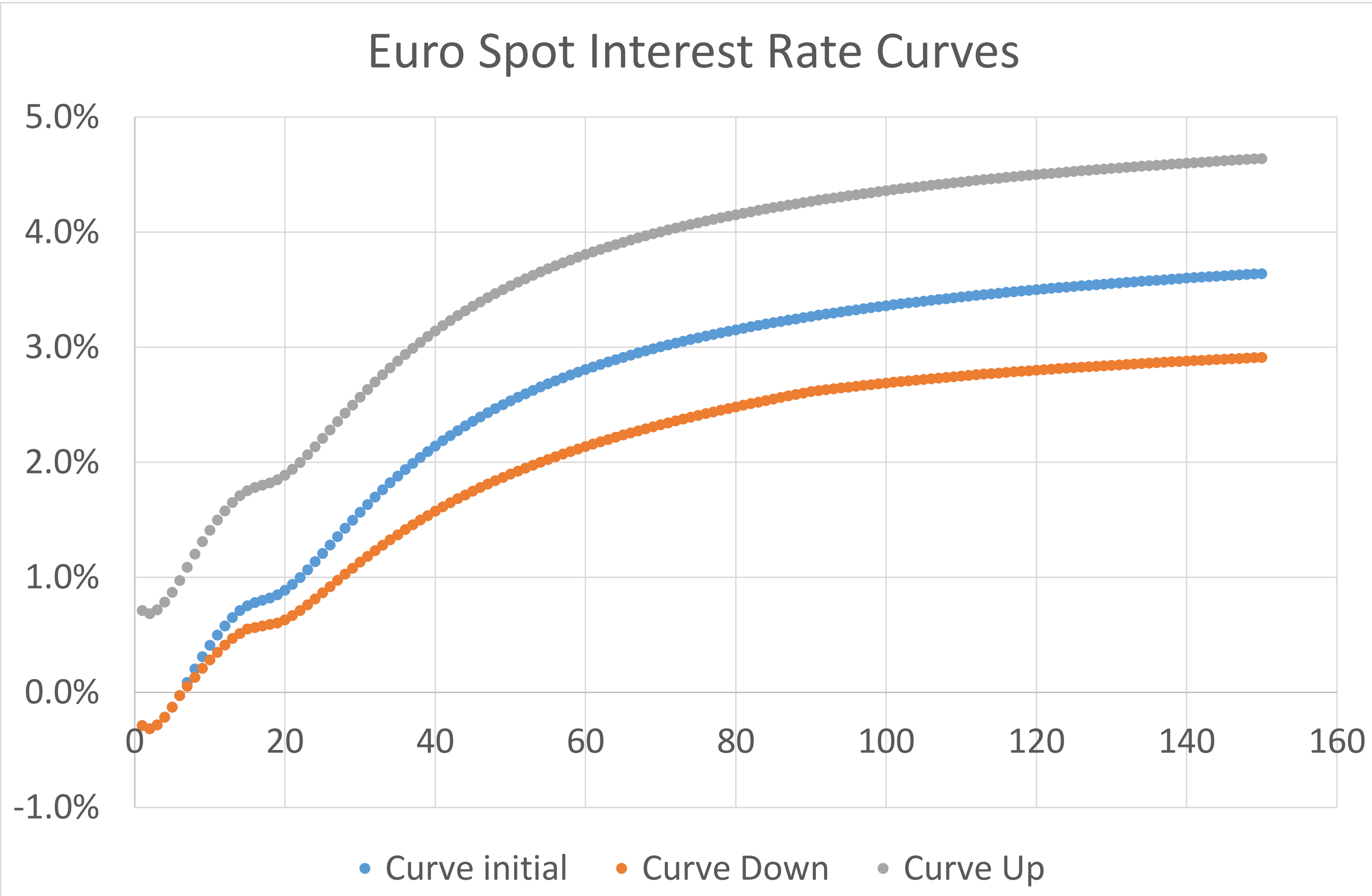
Developed : $SCR_{eq,1} = \Delta_{eq,1} A_{eq,1}$, $\Delta_{eq,1}$ = 39%

Other : $SCR_{eq,2} = \Delta_{eq,2} A_{eq,2}$, $\Delta_{eq,2}$ = 49%

$$SCR_{Mkt,II} = \sqrt{(SCR_{eq,1})^2 + (SCR_{eq,2})^2 + 2\rho_{eq} SCR_{eq,1} SCR_{eq,2}}$$

with ρ_{eq}=0.75

SCR for Interest Rate Risk



Market Risk Assets

Market risk assets

- $A_1 = A_{gov,1}$ = gov. debt EEA countries, with duration $D_{A,1}$
- $A_2 = A_{gov,2}$ = gov. debt other countries, duration $D_{A,2}$
- $A_3 = A_{corp}$ = corporate debt, with duration $D_{A,3}$
- $A_4 = A_{eq}$ = equity
- $A_5 = A_{prop}$ = property

Equity includes: listed stocks in developed and emerging markets, as well as investments in private equity and hedge funds.

Non-market assets are charged in a separate module, dealing with counter-party risk: e.g., cash, mortgages and reinsurance assets

Liabilities

<u>Liability categories</u>

- $L_1 = L_{tprov}$ = technical provisions, with duration $D_{L,1}$
- $L_2 = L_{other}$ = other liabilities, duration $D_{L,2}$

Technical provisions: the value of all the insurance liabilities, using market-based valuation

- Present value of all expected cash flows discounted with swap rates (adjusted for credit risk)

Other liabilities

- For example, short-term debt and deferred tax liabilities

SCR for Interest Rate Risk

I. Interest rate risk

Max loss, after comparing impact of downward (Δ_{rd}) and upward interest rate shocks (Δ_{ru}) on the insurer balance sheet

Simple duration-based approximation, following Höring (2013):

$$SCR_{Mkt,I} = \max\{\ \Delta_{rd}((D_{L,1}L_1 + D_{L,2}L_2) - (D_{A,1}A_1 + D_{A,2}A_2 + D_{A,3}A_3)),$$

$$\Delta_{ru}\big(D_{A,1}A_1 + D_{A,2}A_2 + D_{A,3}A_3\big) - (D_{L,1}L_1 + D_{L,2}L_2))\ \}$$

SCR for Spread and Currency Risk

IV. Credit spread risk:

$$SCR_{Mkt,IV} = \Delta_{gov,2} A_{gov,2} + \Delta_{corp} A_{corp}$$

with the shocks $\Delta_{gov,2}$ and Δ_{corp} depending on the credit rating and duration of the bond portfolios

V. Currency risk:

$$SCR_{Mkt,V} = \Delta_{cur} \sum_{i=1}^{I} f_i A_i$$

with f_i the fraction of asset i invested in foreign currencies, and Δ_{cur}=25%

SCR for Market Risk: Aggregation

SCR for Market Risk:

$$SCR_{Market} = \sqrt{\sum_{k=1}^{K} (SCR_{Mkt,k})^2 + \sum_{k=1}^{K} \sum_{\substack{j=1 \\ j \neq k}}^{K} \rho_{kj} SCR_{Mkt,k} SCR_{Mkt,j}}$$

where $\rho_{kj} = \rho_{Mkt,kj}$ is the correlation between market risk types *k* and *j*, prescribed by the regulator.

I. Interest rate risk : $SCR_{Mkt,I}$
II. Equity risk : $SCR_{Mkt,II}$
III. Property risk : $SCR_{Mkt,III}$
IV. Credit spread risk : $SCR_{Mkt,IV}$
V. Currency risk : $SCR_{Mkt,V}$
VI. Concentration risk : $SCR_{Mkt,VI}$

SCR for Market Risk: Aggregation

The correlations between market risk types, prescribed by the regulator

	Interest	Equity	Property	Spread	Currency
Interest rate risk	1	0.5	0.5	0.5	0.25
Equity risk	0.5	1	0.75	0.75	0.25
Property risk	0.5	0.75	1	0.5	0.25
Spread risk	0.5	0.75	0.5	1	0.25
Currency risk	0.25	0.25	0.25	0.25	1

Note: assuming curve down shock determines interest rate risk, and no concentration risk

SCR for Market Risk: Aggregation

SCR for Market Risk in vector-matrix notation:

$$SCR_{Market} = \sqrt{\sum_{k=1}^{K} \sum_{j=1}^{K} \rho_{kj} SCR_{Mkt,k} SCR_{Mkt,j}} = (\mathbf{s}'\mathbf{R}\mathbf{s})^{½}$$

$\mathbf{s} = (SCR_{Mkt,I}, SCR_{Mkt,II}, \dots, SCR_{Mkt,K})'$

is a $Kx1$ vector with the SCR's for the market risk types

$\mathbf{R}$ is a KxK matrix with the correlation coefficients ρ_{kj} between market risk types *k* and *j*, prescribed by the regulator

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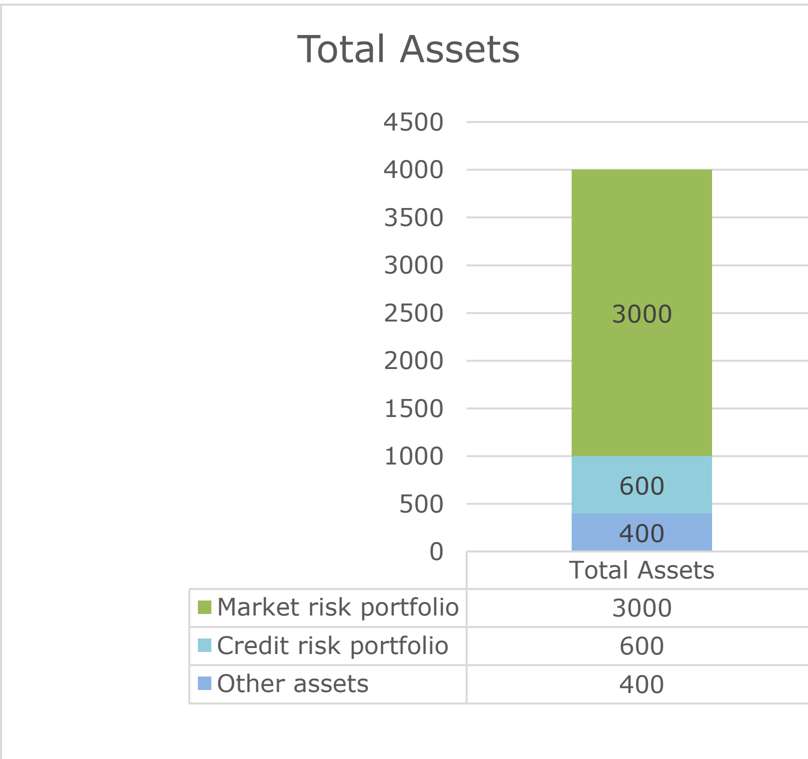
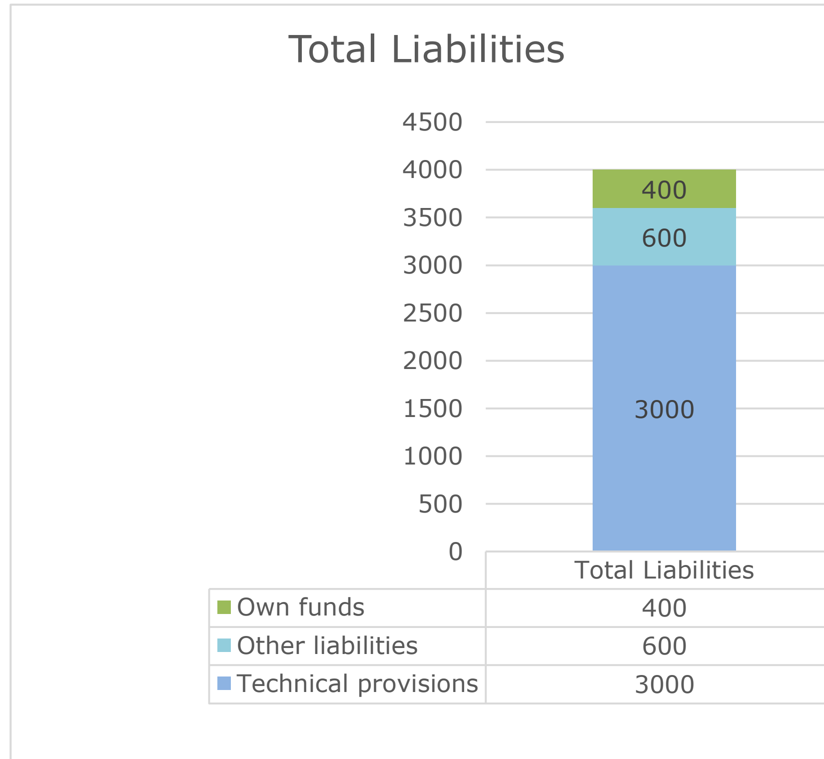
Data from Höring (2013):





Note: all amounts in million euro

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Table 3, Market risk assets

		Weight	Value	Duration	DV01	E[r]
Market risk assets						
	Gov. debt (EEA)	32.0%	960	6.9	0.66	1.5%
	Gov. debt (non-EEA)	8.0%	240	6.9	0.17	1.8%
	Corporate debt	29.5%	885	5.4	0.48	2.4%
	Covered bonds	12.5%	375	6.2	0.23	1.8%
	Global equities	4.5%	135	0	0	4.5%
	Other equities	2.5%	75	0	0	5.5%
	Real estate	11.0%	330	0	0	3.5%
	Total	100.0%	3,000	5.1	1.54	2.3%

Note: all asset values in million euro

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Table 4, Balance sheet

		Weight	Value	Duration	DV01	E[r]
Total assets						
	Total market risk portfolio	75%	3,000	5.1	1.54	2.3%
	Credit risk portfolio	15%	600	4.9	0.29	3.5%
	Other assets	10%	400	0	0	0.3%
	Total	100.0%	4,000	4.6	1.83	2.3%
Total liabilities						
	Technical provisions	75%	3,000	8.9	2.67	3.0%
	Other liabilities	15%	600	0	0	0.3%
	Own funds	10%	400	0	0	-0.3%
	Total	100.0%	4,000	6.7	2.67	2.3%

Note: the liabilities have a longer duration than the assets (6.7 vs. 4.6 year)

1 basis point drop in the swap curve leads to a loss of 0.84 million euro

And expected return on own funds is negative (-0.3%)

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Market risk category		SCR	Sub-SCR	Shock	Value
1. Interest rate risk		112.2			
	Market risk assets		-205.5	-6.8%	3,000
	Credit risk portfolio		-38.9	-6.5%	600
	Technical provisions		356.6	11.9%	3,000
2. Equity risk		66.1			
	Developed equity		40.5	30.0%	135
	Other equity		30.0	40.0%	75
	Gross equity risk		70.5		
	Diversification benefits		-4.4		
3. Property risk		82.5		25.0%	330
4. Credit spread risk		100.9			
	Gov. debt (non-EEA)		6.0	2.5%	240
	Corporate debt		79.2	9.0%	885
	Covered bonds		15.7	4.2%	375
5. Currency risk		0.0		25.0%	0
Gross SCR for market risk		361.7			
	Diversification benefits	-64.3			
Total SCR for market risk		**297.4**			

Optimal Asset Allocation

Objective:

Maximize expected return on own funds (F = A – L),
subject to an upper limit on SCR for market risk (SCR_{Market}^{Max})

$$\max_{\boldsymbol{a}} E[\Delta F(\boldsymbol{a})] = r_f A_0 + \sum_{i=1}^{I} \mu_{A,i} A_i - \sum_{n=1}^{N} \mu_{L,n} L_n$$

subject to $SCR_{Market}(\boldsymbol{a}; L_1, L_2) \leq SCR_{Market}^{Max}$

Note: can borrow/lend at the short-term risk-free rate r_f

Asset Allocation: Assumptions

- Insurance liabilities have longer duration than the assets:
  - $SCR_{Mkt,I} = \Delta_{rd}\big((D_{L,1}L_1 + D_{L,2}L_2) - (D_{A,1}A_1 + D_{A,2}A_2 + D_{A,3}A_3)\big)$

- Within the equity portfolio, the weights of developed equity and other equity are fixed at $\mathrm{w}_{eq,1}$ and $1 - \mathrm{w}_{eq,1}$
  - Can treat equity as a single asset class with invested amount $A_4 = A_{eq}$, and solvency shock: $\Delta_{eq} = w_{eq,1}\Delta_{eq,1} + \left(1 - w_{eq,1}\right)\Delta_{eq,2}$

Asset Allocation: Linear SCR's

Under these assumptions, the SCR's for the market risk types are a linear function of the asset and liability values:

$$\mathbf{s} = \mathbf{V}\boldsymbol{a} + \boldsymbol{c_L}$$

$s = (SCR_{Mkt,I}, SCR_{Mkt,II}, \dots, SCR_{Mkt,K})'$ is a $Kx1$ vector holding the SCR's,
$a = (A_1, A_2, \cdots, A_I)'$ is a $Ix1$ vector with the risky asset amounts.

$$\mathbf{v}_{\boldsymbol{A}}(1) = (-D_{A,1}\Delta_{rd}, -D_{A,2}\Delta_{rd}, -D_{A,3}\Delta_{rd}, 0, 0)$$
$$\mathbf{v}_{\boldsymbol{A}}(2) = \left(0, 0, 0, \Delta_{eq}, 0\right)$$
$$\mathbf{v}_{\boldsymbol{A}}(3) = \left(0, 0, 0, 0, \Delta_{prop}\right)$$
$$\mathbf{v}_{\boldsymbol{A}}(4) = (0, \Delta_{gov,2}, \Delta_{corp}, 0, 0)$$
$$\mathbf{v}_{\boldsymbol{A}}(5) = (\Delta_{cur}f_1, \Delta_{cur}f_2, \Delta_{cur}f_3, \Delta_{cur}f_4, \Delta_{cur}f_5)$$

$$\boldsymbol{c_L} = (\Delta_{rd}\left(D_{L,1}L_1 + D_{L,2}L_2\right), 0, \dots, 0)'$$

Optimal Asset Allocation

Objective:

$$\max_{\boldsymbol{a}} E[\Delta F(\boldsymbol{a})] = r_f A + \boldsymbol{\mu_A}'\boldsymbol{a} - \mu_{L,1} L_1 - \mu_{L,2} L_2$$

subject to $\left((\mathbf{V}\boldsymbol{a} + \boldsymbol{c_L})'\mathbf{R}(\mathbf{V}\boldsymbol{a} + \boldsymbol{c_L})\right)^{½} \leq SCR_{Market}^{Max}$

First-order condition:

$$\boldsymbol{\mu_A} - \lambda \frac{\mathbf{V}'\mathbf{R}(\mathbf{Va} + \boldsymbol{c_L})}{SCR_{Market}} = \mathbf{0}, \qquad \text{for } \lambda \geq 0$$

where $\boldsymbol{\mu_A} = (\mu_{A,1} - r_f, \mu_{A,2} - r_f, \cdots, \mu_{A,I} - r_f)'$ is a vector of expected asset class *excess* returns

Optimal Asset Allocation

Solving the first-order condition:

$$\boldsymbol{a}^* = \frac{SCR_{Market}^{Max}}{\lambda}(\mathbf{V}'\mathbf{R}\mathbf{V})^{-1}\boldsymbol{\mu}_{\boldsymbol{A}} - (\mathbf{V}'\mathbf{R}\mathbf{V})^{-1}\mathbf{V}'\mathbf{R}\boldsymbol{c}_{\boldsymbol{L}}$$
$$= \frac{SCR_{Market}^{Max}}{\lambda}(\mathbf{V}'\mathbf{R}\mathbf{V})^{-1}\boldsymbol{\mu}_{\boldsymbol{A}} - \mathbf{V}^{-1}\boldsymbol{c}_{\boldsymbol{L}}$$

where $\lambda \geq 0$ is a Lagrange multiplier for the SCR constraint.

$$\lambda = \sqrt{\boldsymbol{\mu}_{\boldsymbol{A}}'(\mathbf{V}_{\boldsymbol{A}}'\mathbf{R}\mathbf{V}_{\boldsymbol{A}})^{-\mathbf{1}}\boldsymbol{\mu}_{\boldsymbol{A}}} = RoC_{NoLiab}^*$$

Assumption: $\mathbf{V}'\mathbf{R}\mathbf{V}$ is invertible

Optimal Asset Allocation

Solution:

$$\boldsymbol{a}^* = \left(\frac{SCR_{Market}^{Max}}{RoC_{NoLiab}^*}\right)(\mathbf{V}'\mathbf{R}\mathbf{V})^{-\mathbf{1}}\boldsymbol{\mu}_{\boldsymbol{A}} - \mathbf{V}^{-\mathbf{1}}\boldsymbol{c}_{\boldsymbol{L}}$$

$$= \boldsymbol{a}_{\boldsymbol{NoLiab}}^* + \boldsymbol{a}_{\boldsymbol{Hedge}}^*$$

where

$\boldsymbol{a}_{\boldsymbol{NoLiab}}^* = \left(\frac{SCR_{Market}^{Max}}{RoC_{NoLiab}^*}\right)(\mathbf{V}'\mathbf{R}\mathbf{V})^{-\mathbf{1}}\boldsymbol{\mu}_{\boldsymbol{A}}$: optimal asset-only portfolio

$\boldsymbol{a}_{\boldsymbol{Hedge}}^* = -\mathbf{V}^{-\mathbf{1}}\boldsymbol{c}_{\boldsymbol{L}}$: liability-hedge portfolio

Optimal Asset Allocation: Three-Fund Separation

Solution:

$$\boldsymbol{a}^* = \boldsymbol{a}^*_{NoLiab} + \boldsymbol{a}^*_{Hedge}$$

The optimal asset allocation consists of:

(1) A liability-hedge portfolio that reduces the SCR for interest-rate risk to zero (duration matching)

(2) An optimal asset-only portfolio, optimal for an insurer without liabilities

(3) Cash

Optimal Asset Allocation: Condition

For the derivation of the optimal asset allocation we had to assume that the matrix $\mathbf{V}'\mathbf{R}\mathbf{V}$ is invertible

- Because $\mathbf{R}$ is a correlation matrix,
  this means that the KxI matrix $\mathbf{V}$ <u>has to be of full rank</u>
- $\mathbf{V}$ contains the asset charges for the K risk market types

<u>The condition means:</u>

- No more than K risky assets in the optimization: $I \leq K$
- For each type of market risk $k = 1, 2, \ldots, K$ in the SF,
  at least one risky asset needs to have exposure to it
- The exposures of the risky assets to the market risk types contained in V should not be linearly dependent

Optimal Asset Allocation: Lessons

- The KxI matrix $\mathbf{V}$ has to be of full rank
- $\mathbf{V}$ contains the asset charges for the K risk market types

The condition means:

- Suppose the insurer has exposure to 5 risk types,
  than the asset allocation can only have max 5 asset classes
- If not, we will have redundant assets
  - And potentially arbitrage opportunities if shorting is allowed

Optimal Asset Allocation: Lessons

- The Solvency II standard formula for market risk is a simplified model that does not distinguish the risk of assets *within* the same market risk type k well

For example, suppose insurer can choose between:

1. The MSCI World: a well-diversified portfolio of 1,637 stocks spread over 23 developed markets
   - Suppose expected return is 7% per annum
2. The MSCI Belgium: a portfolio of 10 stocks from one developed country, Belgium
   - Suppose the expected return is 8% per annum

As the risk charge is equal ($\Delta_{eq,1} = 39\%$) for all developed equity, the optimizer would prefer MSCI Belgium over MSCI World

Optimal Asset Allocation: Lessons

- The Solvency II standard formula for market risk is a simplified model that does not distinguish the risk of assets *within* the same market risk type k well

- In the absence of an internal risk model, only well-diversified portfolios should be considered for getting exposure to the market risk types of the standard formula
  - For example, the MSCI World for developed equity,
  - A broadly diversified corporate bond portfolio for spread risk, etc.

- After that, the standard formula can still be applied for risk aggregation and risk allocation *between* the risk types

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- Initial asset allocation

Market risk portfolio	Value	Weight	E[r]					
Equity portfolio	210	7.0%	4.9%					
Real estate	330	11.0%	3.5%					
Gov bonds EEA	960	32.0%	1.5%					
Gov bonds non-EEA	240	8.0%	1.8%					
Corporate debt	885	29.5%	2.4%					
Covered bonds	375	12.5%	1.8%					
Treasury bills EEA	0	0.0%	0.3%					
Total portfolio	**3,000**	**100%**	**2.3%**					
Return/risk trade-off			Dollar duration (DV01)			Leverage of assets		
E[Increase own funds]	-1.3		Assets	1.83		Long	4,000	
SCR market risk	297.4		Liabilities	2.67		Short	0	
Solvency ratio f_M	135%		Gap	0.84		Leverage	1.0	
Return on SCR (RoC)	-0.5%							

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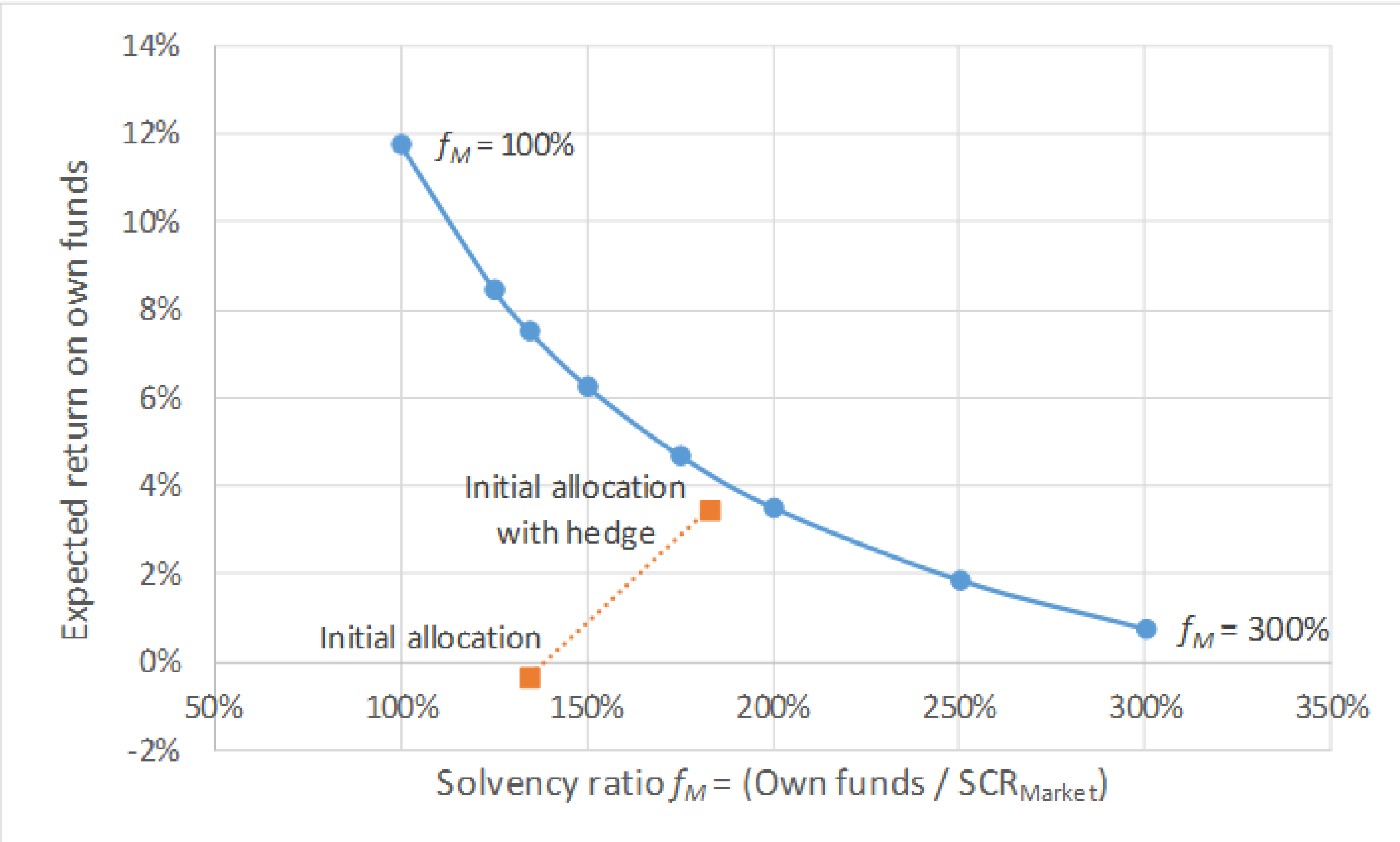
- Asset allocation with liability hedge

Market risk portfolio	**Value**	**Weight**	**E[r]**					
Equity portfolio	210	7.0%	4.9%					
Real estate	330	11.0%	3.5%					
Gov bonds EEA	2,177	72.6%	1.5%					
Gov bonds non-EEA	240	8.0%	1.8%					
Corporate debt	885	29.5%	2.4%					
Covered bonds	375	12.5%	1.8%					
Treasury bills EEA	-1,217	-40.6%	0.3%					
Total portfolio	**3,000**	**100%**	**2.8%**					
Return/risk trade-off			Dollar duration (DV01)			Leverage of assets		
E[Increase own funds]	13.9		Assets	2.67		Long	5,217	
SCR market risk	218.8		Liabilities	2.67		Short	-1,217	
Solvency ratio f_M	183%		Gap	0		Leverage	1.3	
Return on SCR (RoC)	6.3%							

Efficient Frontier



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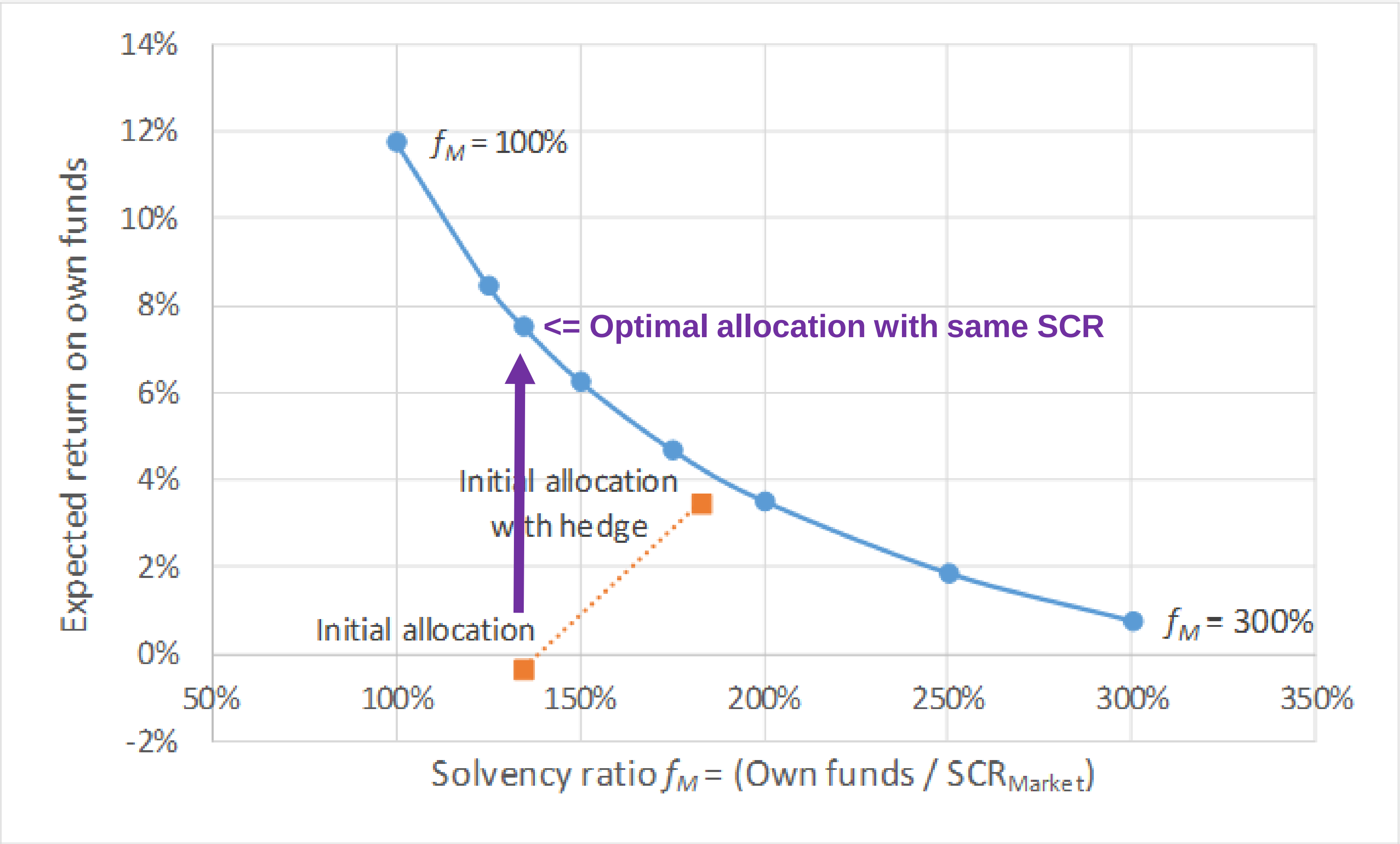
- Optimal asset allocation

Market risk portfolio	**Value**	**Weight**	**E[r]**					
Equity portfolio	367	12.2%	4.9%					
Real estate	213	7.1%	3.5%					
Gov bonds EEA	4,377	145.9%	1.5%					
Gov bonds non-EEA	0	0.0%	1.8%					
Corporate debt	634	21.1%	2.4%					
Covered bonds	0	0.0%	1.8%					
Treasury bills EEA	-2,592	-86.4%	0.3%					
Total portfolio	**3,000**	**100%**	**3.3%**					
Return/risk trade-off			Dollar duration (DV01)			Leverage of assets		
E[Increase own funds]	30.2		Assets	3.65		Long	6,592	
SCR market risk	297.4		Liabilities	2.67		Short	-2,592	
Solvency ratio f_M	135%		Gap	-0.98		Leverage	1.6	
Return on SCR (RoC)	10.2%							

Efficient Frontier



Ratio of Expected Return to Marginal Risk

First-order condition:

$$\frac{\left(\mu_{i,A} - r_f\right)}{mSCR_{A,i}^{Mkt}} = \lambda, \qquad \text{for } i = 1, 2, \ldots, I$$

Where

$$mSCR_{A,i}^{Mkt} = \frac{\partial SCR_{Market}}{\partial A_{\mathrm{i}}}$$

$mSCR_{A,i}^{Mkt}$ is the marginal increase in SCR (risk) when the investment in asset class *i* is increased by 1 unit (a small amount).

Representative European Life Insurance Company

- Initial asset allocation

Market risk portfolio	**Value**	**Weight**	**E[r]**	**mSCR**	**$E[r_e]$ /mSCR**
Equity portfolio	210	7.0%	4.9%	0.27	**0.17**
Real estate	330	11.0%	3.5%	0.20	**0.16**
Gov bonds EEA	960	32.0%	1.5%	-0.07	**-0.17**
Gov bonds non-EEA	240	8.0%	1.8%	-0.05	**-0.29**
Corporate debt	885	29.5%	2.4%	0.02	**1.26**
Covered bonds	375	12.5%	1.8%	-0.03	**-0.48**
Treasury bills EEA	0	0.0%	0.3%	0.00	---
Total portfolio	**3,000**	**100%**	**2.3%**	**0.01**	**1.37**

Return/risk trade-off		Dollar duration (DV01)		Leverage of assets	
E[Increase own funds]	-1.3	Assets	1.83	Long	4,000
SCR market risk	297.4	Liabilities	2.67	Short	0
Solvency ratio f_M	135%	Gap	0.84	Leverage	1.0
Return on SCR (RoC)	-0.5%				

Representative European Life Insurance Company

- Optimal asset allocation

Market risk portfolio	Value	Weight	E[r]	mSCR	$E[r_e]$ /mSCR		
Equity portfolio	367	12.2%	4.9%	0.28	**0.165**		
Real estate	213	7.1%	3.5%	0.20	**0.165**		
Gov bonds EEA	4,377	145.9%	1.5%	0.08	**0.165**		
Gov bonds non-EEA	0	0.0%	1.8%	0.10	**0.157**		
Corporate debt	634	21.1%	2.4%	0.13	**0.165**		
Covered bonds	0	0.0%	1.8%	0.10	**0.148**		
Treasury bills EEA	-2,592	-86.4%	0.3%	0.00	---		
Total portfolio	**3,000**	**100%**	**3.3%**	**0.19**	**0.165**		
Return/risk trade-off			Dollar duration (DV01)			Leverage of assets	
E[Increase own funds]	30.2		Assets	3.65		Long	6,592
SCR market risk	297.4		Liabilities	2.67		Short	-2,592
Solvency ratio f_M	135%		Gap	-0.98		Leverage	1.6
Return on SCR (RoC)	10.2%						

Extensions

- Portfolio optimization is very sensitive to expected asset returns
  - Which are notoriously hard to forecast accurately

Potential methods to deal with this

- Apply robust optimization, or
- Bayesian techniques for parameter uncertainty

Thank you

Q & A