Phatra

Liquidity-Adjusted VaR

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การประชุมวิชาการ “ศาสตราจารย์สังเวียน อินทรวิชัย ด้านตลาดการเงินไทย”

14 พฤศจิกายน 2560

A member of the
Kiatnakin Phatra Financial Group

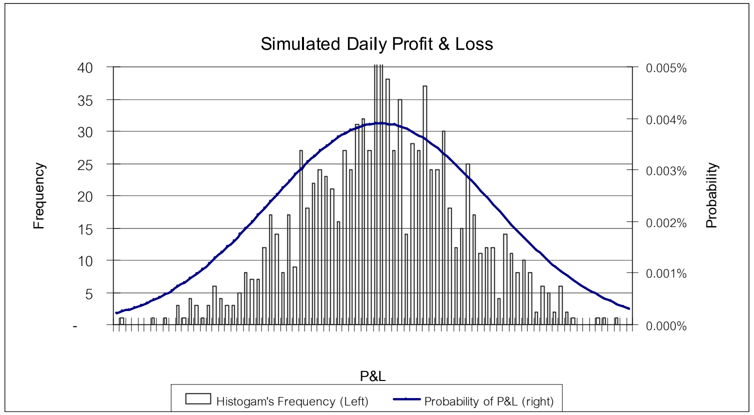
Outlines

Re-visit standard VaR model/calculation – Analytical VaR

- One among 3 approaches (Analytical VaR, Historical VaR and Monte Carlo simulation VaR), derived from closed form solutions
- The possible values of the market variables can be captured by Variance-Covariance Matrix (VCM) of the market returns, which can be computed by several approaches, such as simple, GARCH, EWMA volatility models.
- Historical market data (i.e. security prices) are required to calculate volatility of security or portfolio.

Re-visit standard VaR model/calculation – Analytical VaR

- Analytical VaR (Variance-Covariance)



$$\boxed{VaR_{1-\alpha} = -Z_{\alpha}\sigma_{p}}$$

$$\sigma_P = \sqrt{\begin{bmatrix} W_1 & \dots & W_n \end{bmatrix} \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{bmatrix} \begin{bmatrix} W_1 \\ \vdots \\ W_n \end{bmatrix}}$$

Re-visit standard VaR model/calculation – Analytical VaR

Confidential level of VaR is based on Z score

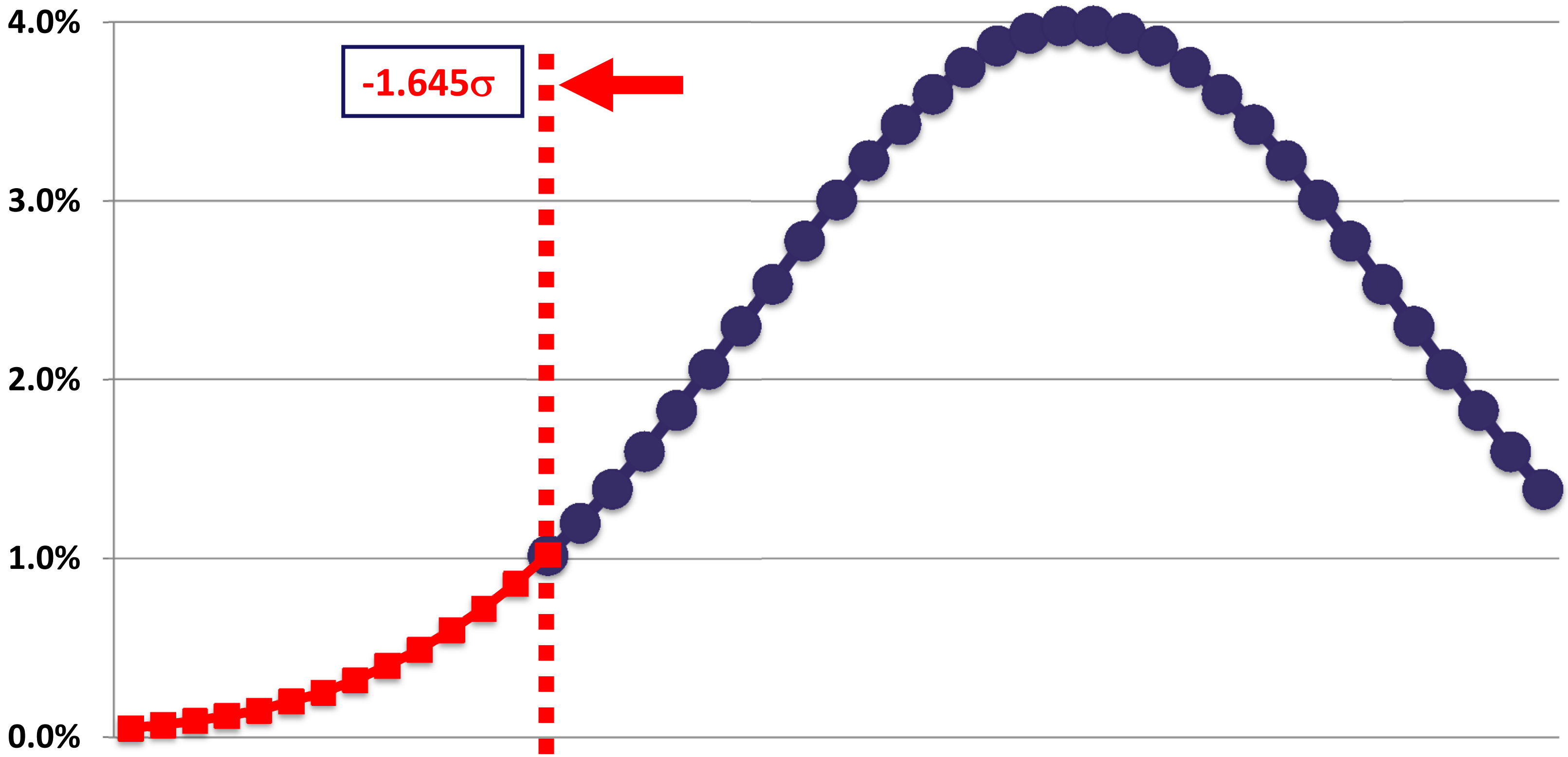
$$VaR_{1-\alpha} = -z_{\alpha}\,\sigma$$

$$VaR_{95\%} = -(-1.645)\sigma$$

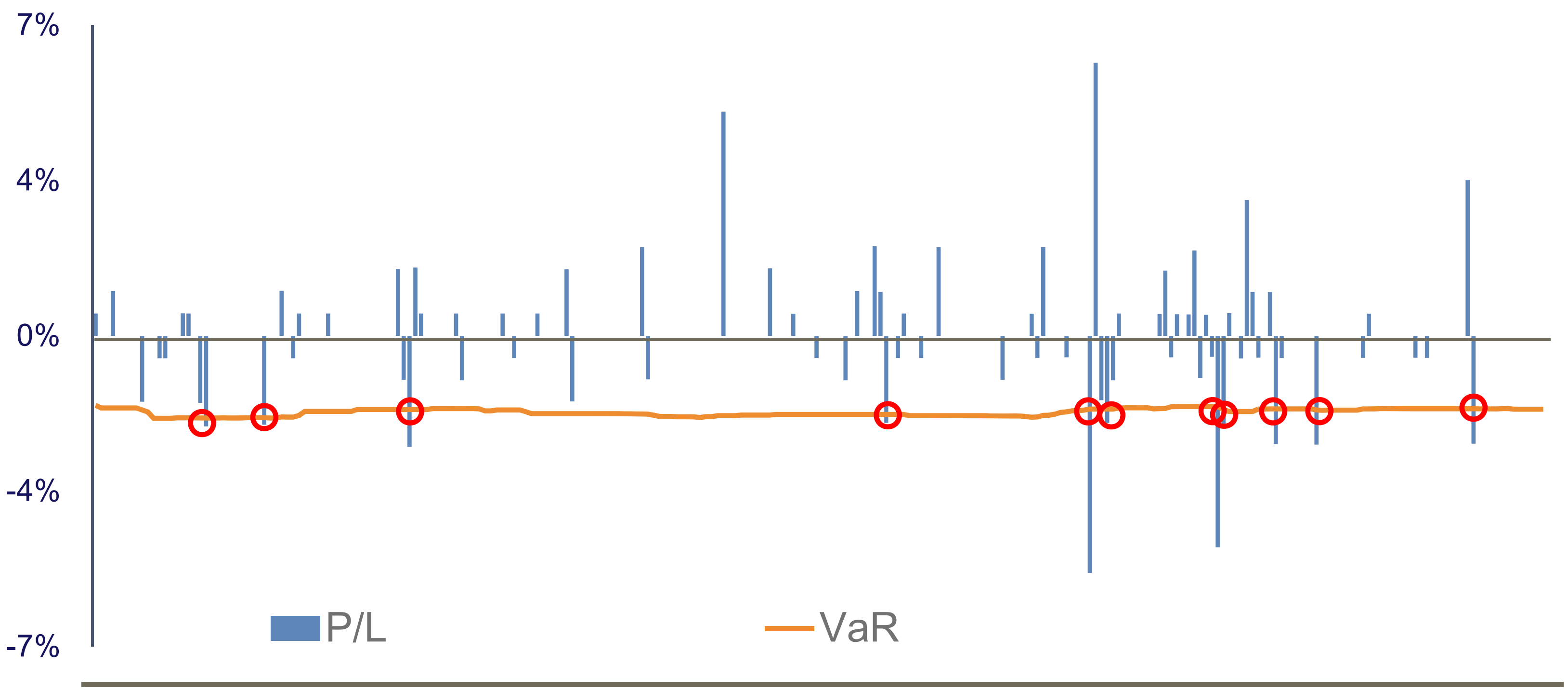
$$VaR_{99\%} = -(-2.33)\,\sigma$$

Re-visit standard VaR model/calculation – Analytical VaR

One-Period VaR (T=1) and 95% confidence (5% significance)

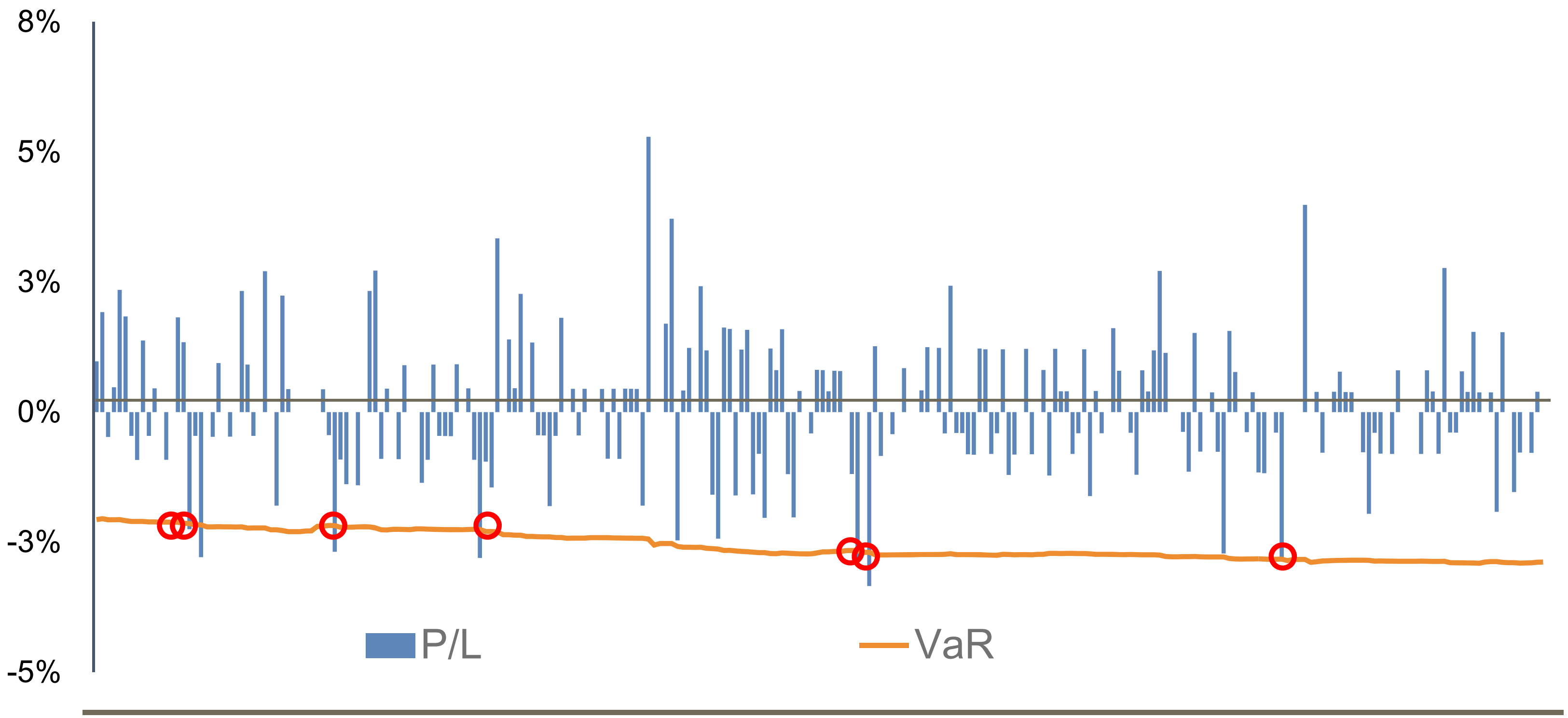


VaR back testing results - WACOAL



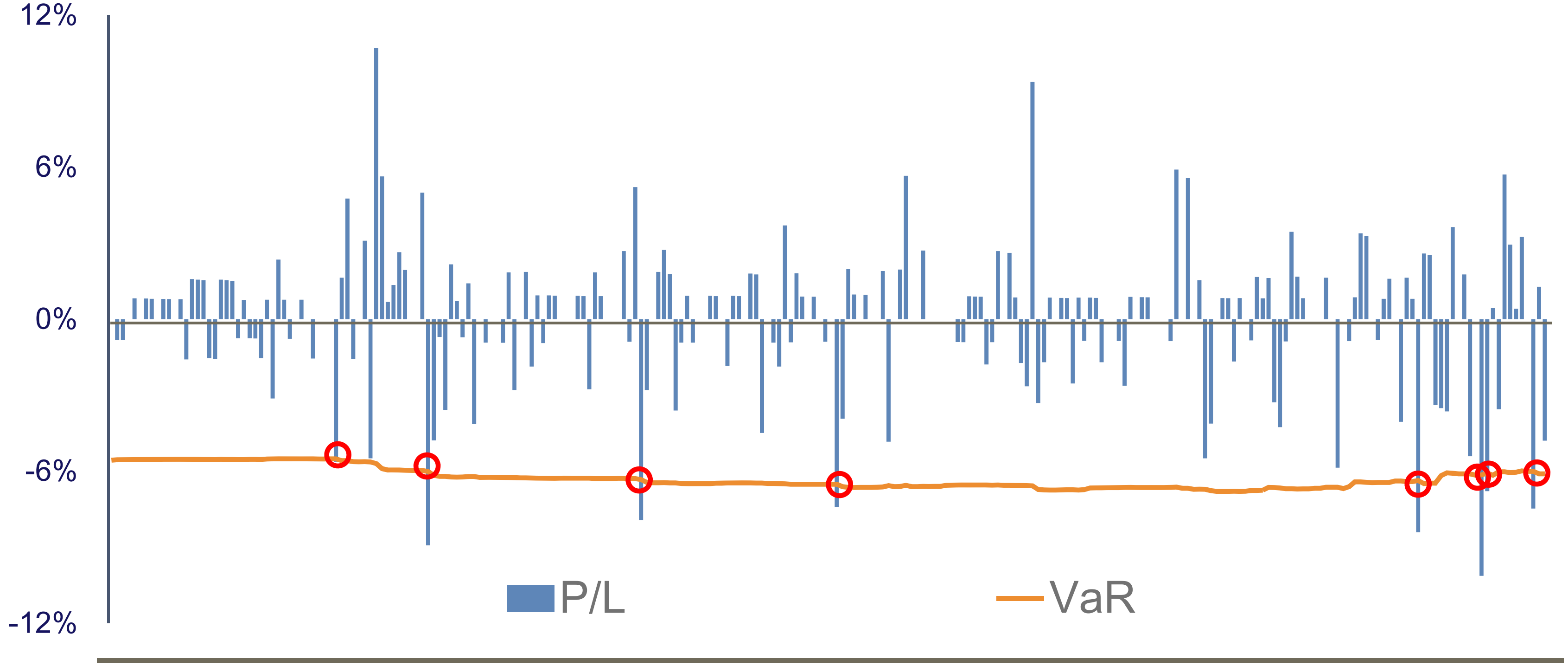
Data from 17/07/16 to 17/07/17

VaR back testing results - APCO



Data from 17/07/16 to 17/07/17

VaR back testing results - M

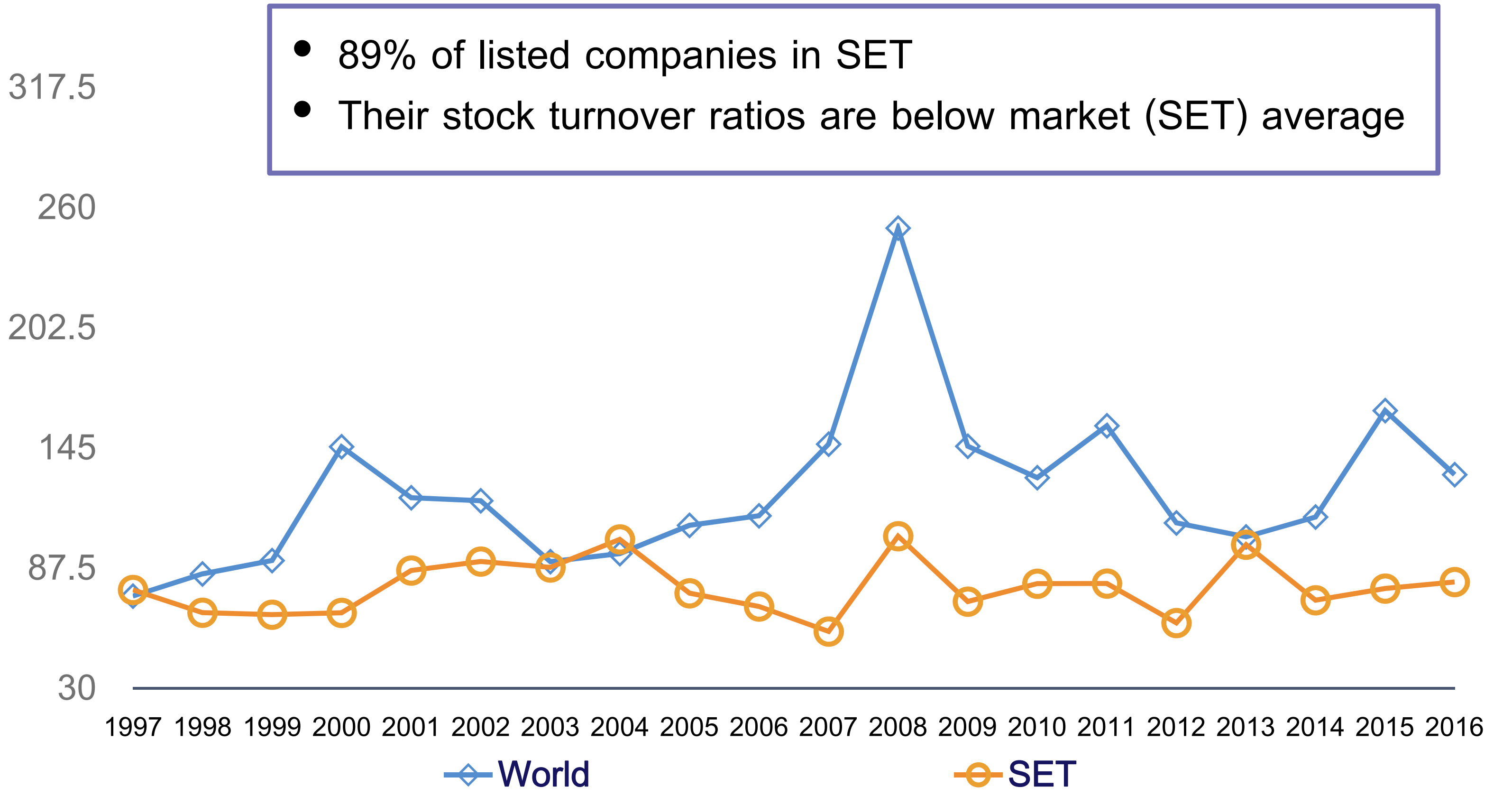


Data from 17/07/16 to 17/07/17

Issues of Analytical VaR model/calculation

- Market data/prices of low liquid stock(s) are used to calculate VaR .
- Market risk of low liquidity security/portfolio is likely underestimated.
- Required to apply scaling factor according to BIS framework or re-visit VaR model.

Comparison of Stock market turnover ratios



Source : Worldbank

Incorporating (i)liquidity to VaR Model

- Standard measures to gauge illiquidity of security

Liquidity Proxy for Stock

Bid-Ask spread	Stock Turnover	Trading Volume	Trading Value

Incorporating (i)liquidity to VaR Model - Market liquidity risk

Endogenous liquidity risk

The result of market characteristic; it is common to all market players and unaffected by the actions of any one participant

Exogenous liquidity risk

Specific to the position in the market and varies across market participants. The exposure of anyone participant is affected by the actions of that participant

Bangia et al. (1998)

Incorporating (i)liquidity to VaR Model

Bangia et al (1999)	• used Bid-Ask spread as a Cost of liquidity to incorporate into VaR
Francios-Heude and Van Wynendaele (2001)	• enhanced Bangia et al. (1999) model by relaxing the Bid-Ask spread distribution and random behavior assumptions
Jarrow and Subramanian (1997)	• proposed the model that incorporate time to liquidation and the liquidity discount into VaR

Jarrow and Subramanian (1997)

- Used liquidity discount as a proxy for liquidity
- The model include both Endogenous and Exogenous liquidity risks

$$\boxed{LVaR_{JS} = \rho.S.\left[\mu.\left\{E(\Delta S) + E\left(\ln[c(S)]\right)\right\} - CI\left(\sigma_E.\sqrt{E(\Delta S)} + |\mu|.\sigma_{\Delta S} + \sigma_{\ln[c(S)]}\right)\right]}$$

Jarrow and Subramanian (1997)

$$\boxed{LVaR_{JS} = \rho . S . \left[\mu . \left\{ E(\Delta S) + E\left(\ln[c(S)] \right) \right\} - CI \left(\sigma_E . \sqrt{E(\Delta S)} + |\mu| . \sigma_{\Delta S} + \sigma_{\ln[c(S)]} \right) \right]}$$

Where

ρ	is the quantity of equity purchased (or short-sold)
S	is the equity price (hence $\rho \times S$ is essentially the nominal amount invested: ie quantity × price)
μ	is the average portfolio return
$E(\Delta S)$	is the expected value of the time horizon to liquidation
$c(S)$	is the liquidity discount – ie the difference between the market value of a trader‘s position and its value when it is ultimately liquidated
CI	is the confidence interval
$\sigma_{\Delta S}$	is the volatility of the time horizon to the position’s liquidation
σ_E	is the equity return volatility and
$\sigma_{\ln[c(S)]}$	is the volatility of the natural logarithm of the liquidity discount.

Jarrow and Subramanian (1997) and Botha (2008)

- Modify LVaR model of Jarrow and Subramanian (1997)
- Apply Ad-hoc approach (JP Morgan/Reuters : 1996)
  - Assume expected return = 0
- Utilize Price drop function parameter (Lau and Kwok : 2001) to determine liquidity discount

Jarrow and Subramanian (1997) and Botha (2008)

Liquidity adjusted VaR

$$LVaR^i_\propto = (\rho \times S_i)\left|\{\mu\{E(\Delta S_i) + E(ln[c(s_i)])\} + Z_\propto\left(\sigma_{e_i}\sqrt{E(\Delta S_i)} + |\mu|\sigma_{\Delta S_i} + \sigma_{ln|c(s_i)|}\right)\right|$$

Assume μ = 0 (JP. morgan/Reuters, 1996)

$$LVaR^i_\propto = Z_\propto\left(\sigma_{e_i}\sqrt{E(\Delta S_i)} + \sigma_{ln|c(s_i)|}\right)$$

$$\Delta S_i = \frac{|S_i|}{\theta\mu_{vol}}$$

where

θ is percentage of positions' liquidated

μ_{vol} is Average 3-month trading volume

$$c(s_i) = p\alpha(s_i)\exp(\mu\Delta S_i)$$

$$\alpha(s_i) = \frac{0.5}{1 - 0.5e^{-as_i}}$$

Lau and Kwok (2001)

Jarrow and Subramanian (1997) and Botha (2008)

Liquidity adjusted VaR

Price-drop function is a pre-deterministic function which decreasing to S, and when S = 0 then alpha(S) = 1

$$\boxed{\alpha(s_i) = \frac{0.5}{1 - 0.5e^{-as_i}}}$$

where

s_i is number of shares liquidated

a is instantaneous price-drop parameters

a = 0.02

Lan and Kwok (2001) studied the optimal liquidating strategy under the Jarrow and Subramanian (1997) model and assumptions.

'Trader will always use liquidation strategy that gives the maximum cash'

Jarrow and Subramanian (1997) and Botha (2008)

Portfolio LVaR

Portfolio parametric VaR (JP. Morgan/Reuters, 1996)

$$VaR_{\propto}^{port} = Z_{\propto}\sqrt{(w_1 \cdots w_n)\begin{pmatrix} \sigma_1^2 & \cdots & \sigma_1\sigma_n\rho_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_n\sigma_1\rho_{n1} & \cdots & \sigma_n^2 \end{pmatrix}\begin{pmatrix} w_i \\ \vdots \\ w_n \end{pmatrix}}$$

$$VaR_{\propto}^{port} = Z_{\propto}\sqrt{(w_1\sigma_1, \cdots, w_n\sigma_n)\begin{pmatrix} 1 & \cdots & \rho_{1n} \\ \vdots & \ddots & \vdots \\ \rho_{n1} & \cdots & 1 \end{pmatrix}\begin{pmatrix} w_i\sigma_1 \\ \vdots \\ w_n\sigma_n \end{pmatrix}}$$

$$\boxed{VaR_{\propto}^{port} = \sqrt{(w_1Z_{\propto}\sigma_1, \cdots, w_nZ_{\propto}\sigma_n)\begin{pmatrix} 1 & \cdots & \rho_{1n} \\ \vdots & \ddots & \vdots \\ \rho_{n1} & \cdots & 1 \end{pmatrix}\begin{pmatrix} w_iZ_{\propto}\sigma_1 \\ \vdots \\ w_nZ_{\propto}\sigma_n \end{pmatrix}}}$$

Jarrow and Subramanian (1997) and Botha (2008)

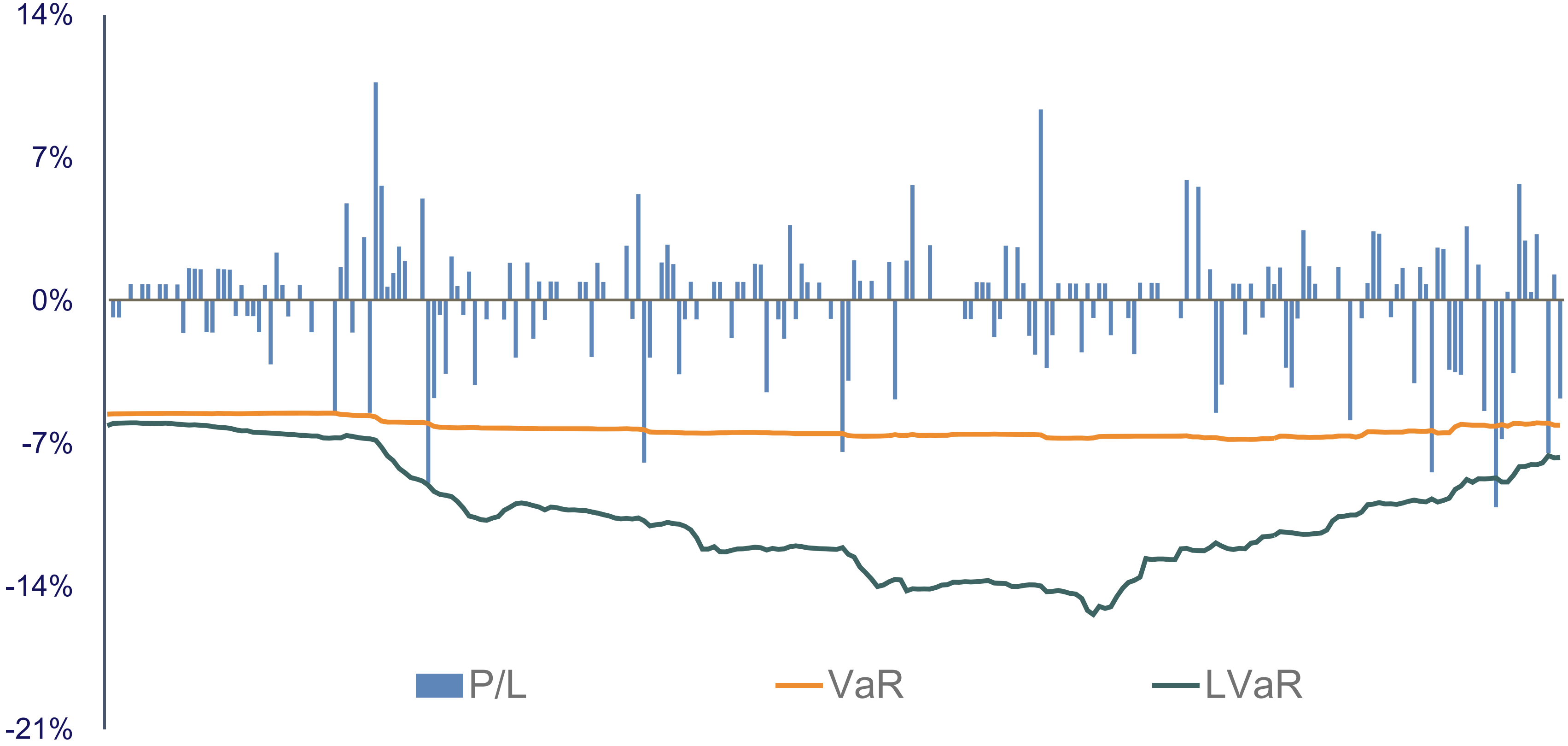
Portfolio LVaR

$$VaR_{\propto}^{port} = \sqrt{(VaR_1, \cdots, VaR_n)\begin{pmatrix} 1 & \cdots & \rho_{1n} \\ \vdots & \ddots & \vdots \\ \rho_{n1} & \cdots & 1 \end{pmatrix}\begin{pmatrix} VaR_1 \\ \vdots \\ VaR_n \end{pmatrix}}$$

Botha (2008) concluded that Portfolio LVaR can be written as

$$\boxed{LVaR_{\propto}^{port} = \sqrt{(LVaR_1, \cdots, LVaR_n)\begin{pmatrix} 1 & \cdots & \rho_{1n} \\ \vdots & \ddots & \vdots \\ \rho_{n1} & \cdots & 1 \end{pmatrix}\begin{pmatrix} LVaR_1 \\ \vdots \\ LVaR_n \end{pmatrix}}}$$

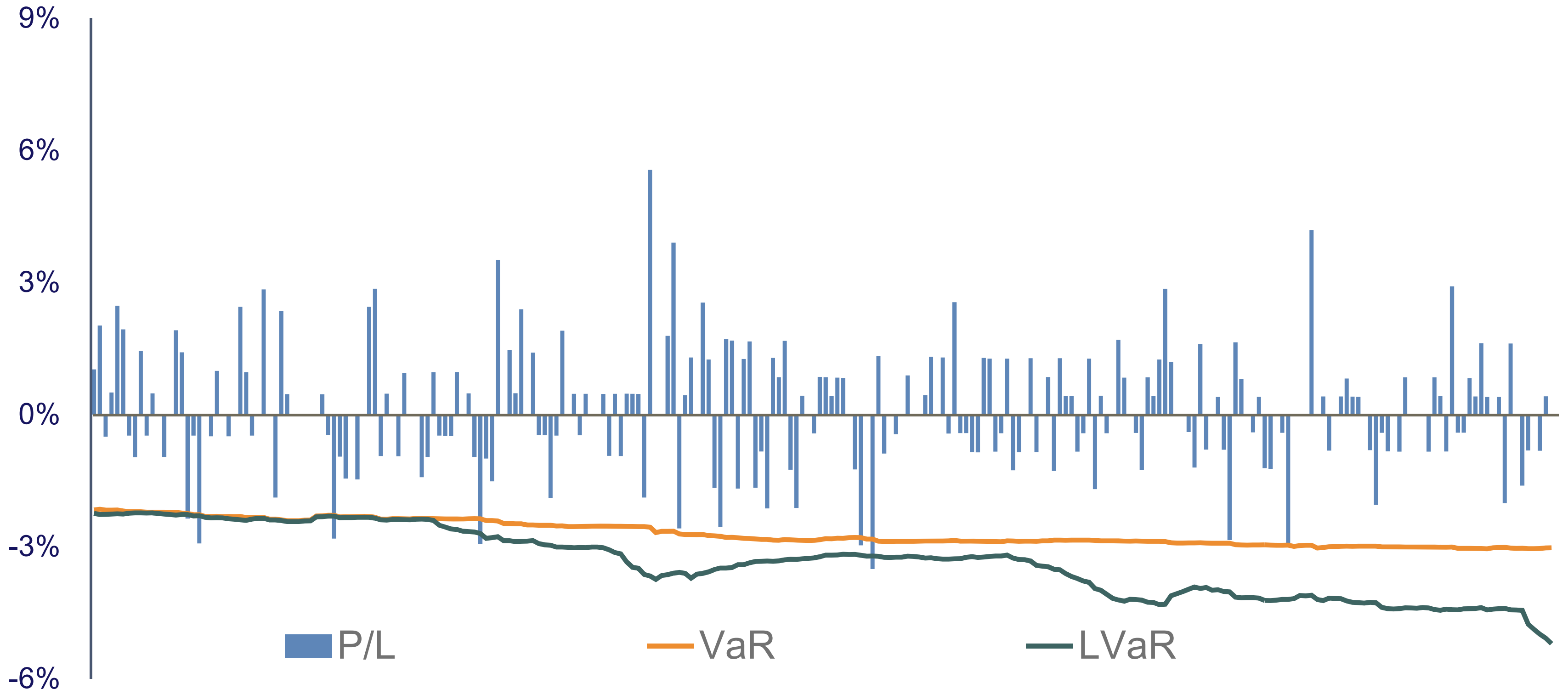
Backtesting LVaR - *APCO*



Suebsak (2017)

Data from 17/07/16 to 17/07/17

Backtesting LVaR - *M*



Suebsak (2017)

Data from17/07/16 to 17/07/17

Backtesting for different levels of liquidity portfolios

High Liquid Portfolio

- 500 MTHB
- 1,500 MTHB
- 2,000 MTHB
- 10,000 MTHB

30 High liquid stocks (SET)

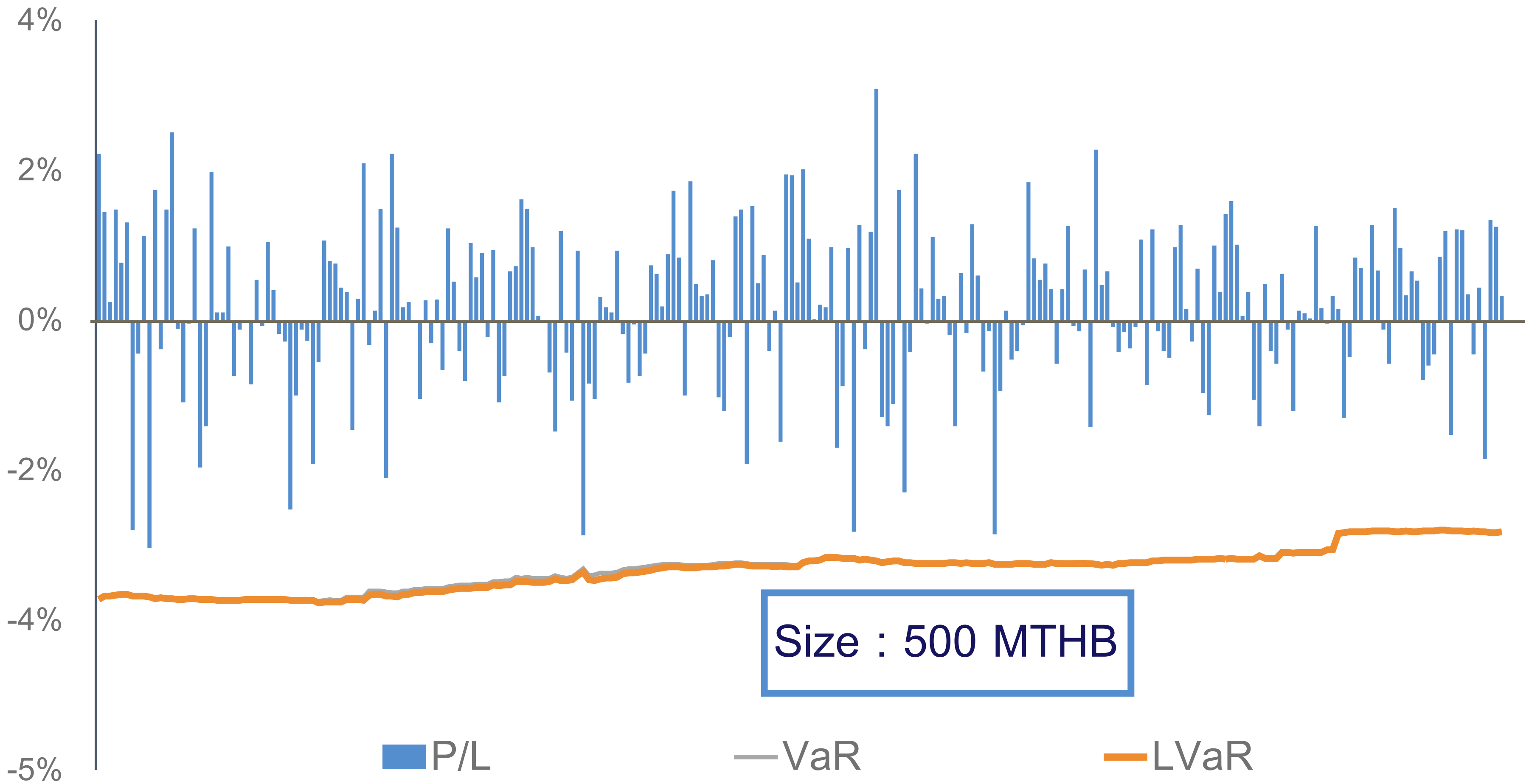
Low Liquid Portfolio

- 2.5 MTHB
- 20 MTHB
- 100 MTHB
- 1,000 MTHB

30 Low liquid stocks (SET)

Backtesting for different levels of liquidity portfolios

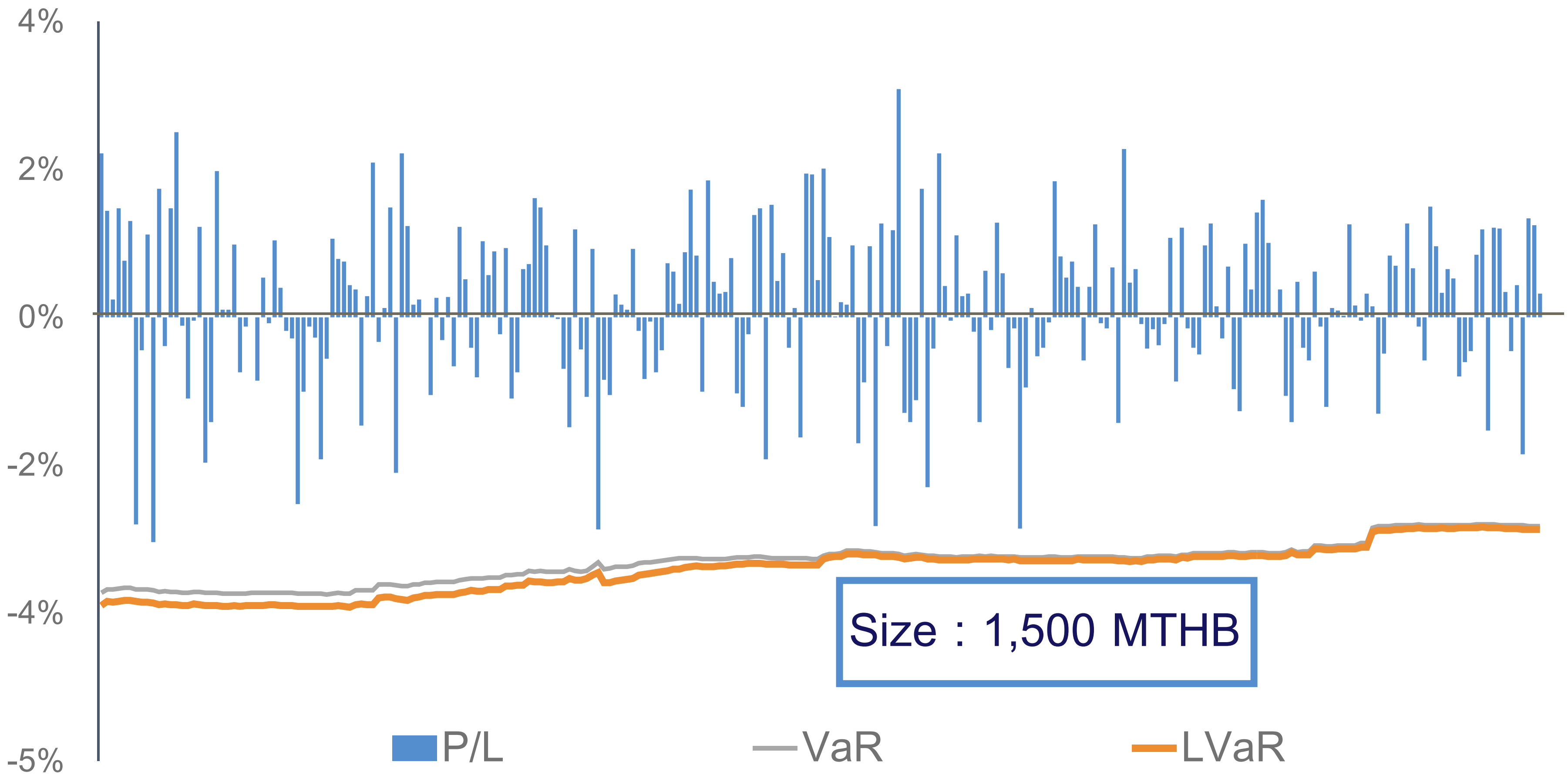
High Liquid Portfolio



Suebsak (2017)

Backtesting for different levels of liquidity portfolios

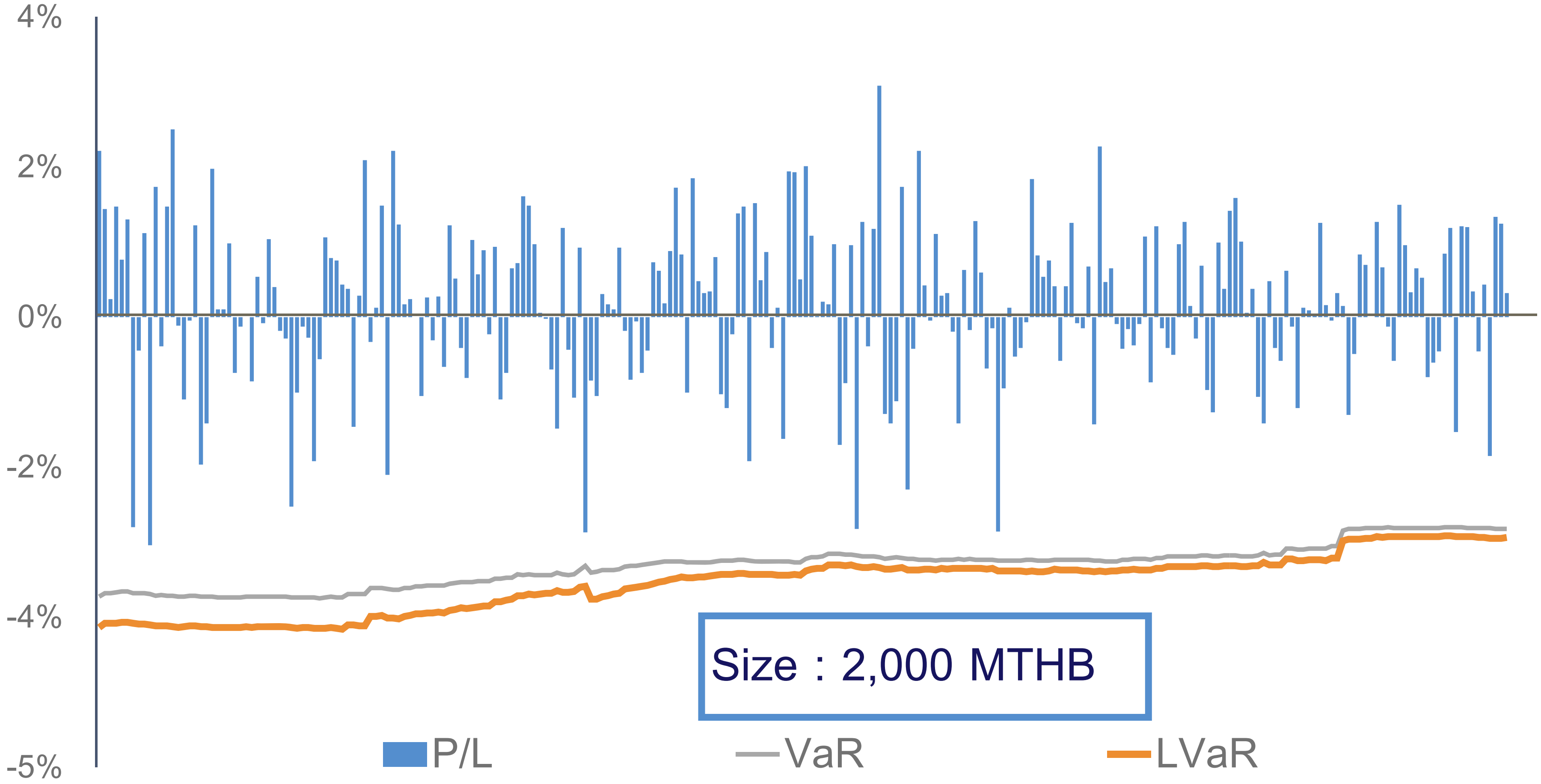
High Liquid Portfolio



Suebsak (2017)

Backtesting for different levels of liquidity portfolios

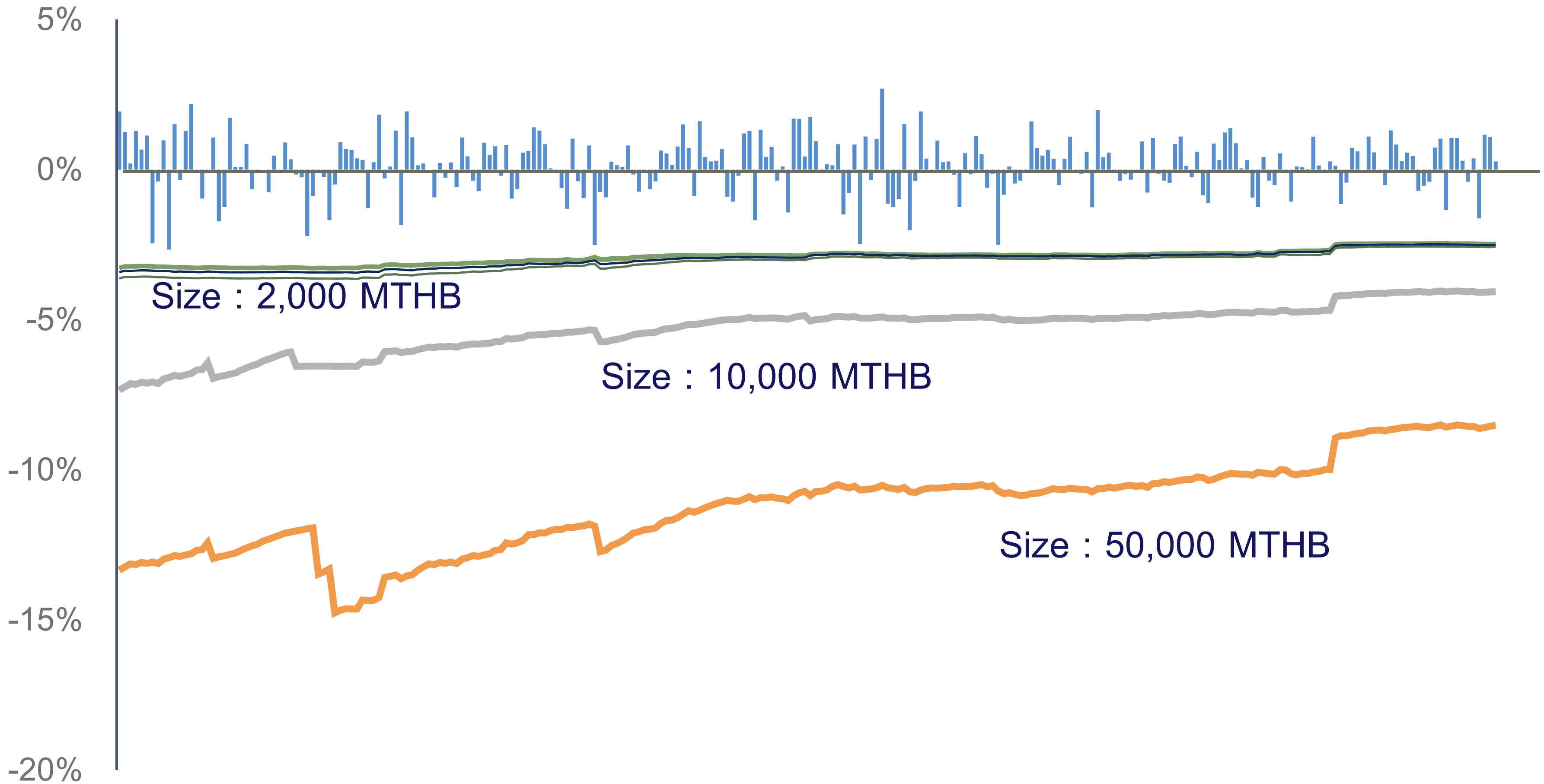
High Liquid Portfolio



Suebsak (2017)

Backtesting for different levels of liquidity portfolios

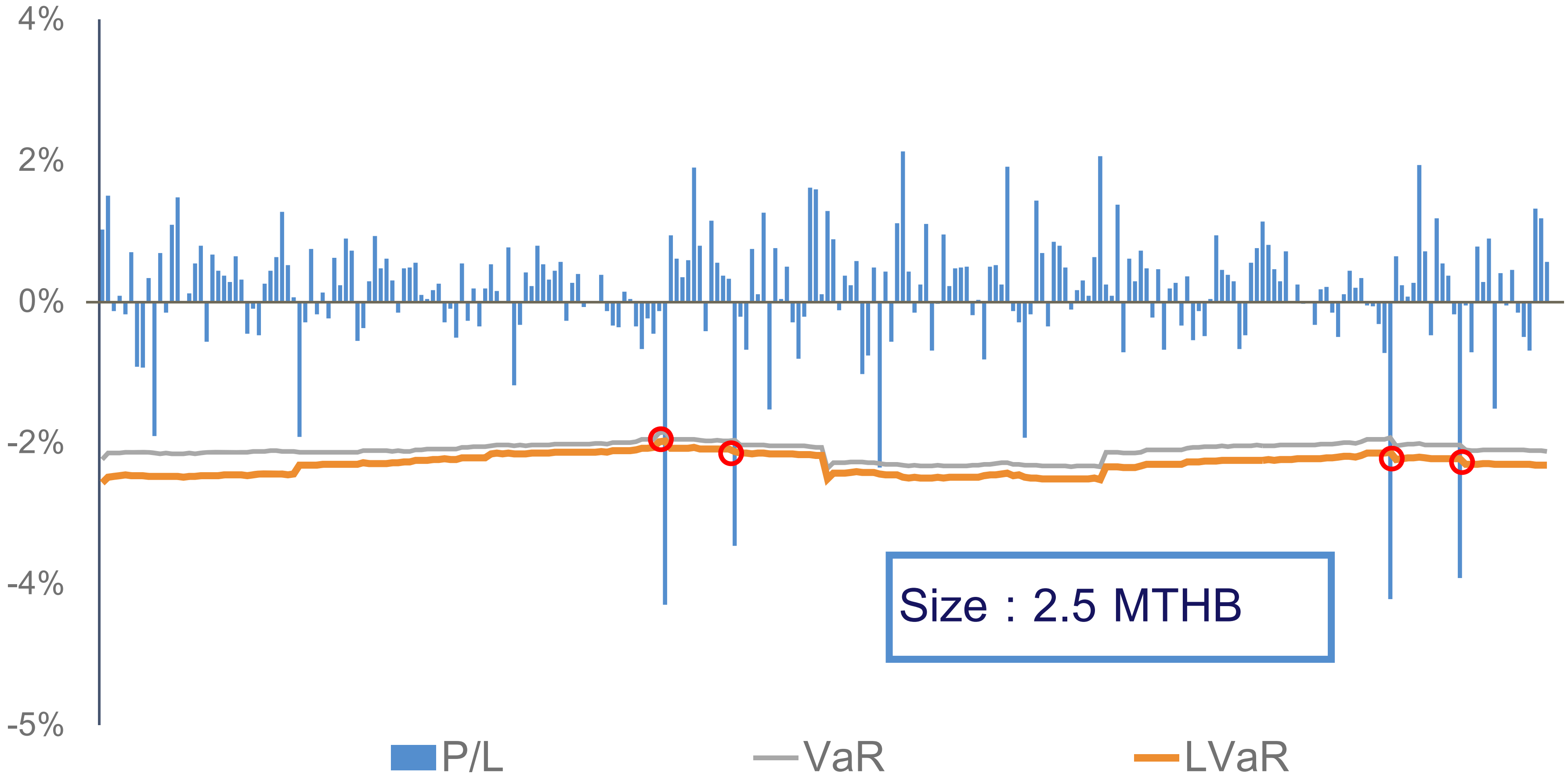
High Liquid Portfolio



Suebsak (2017)

Backtesting for different levels of liquidity portfolios

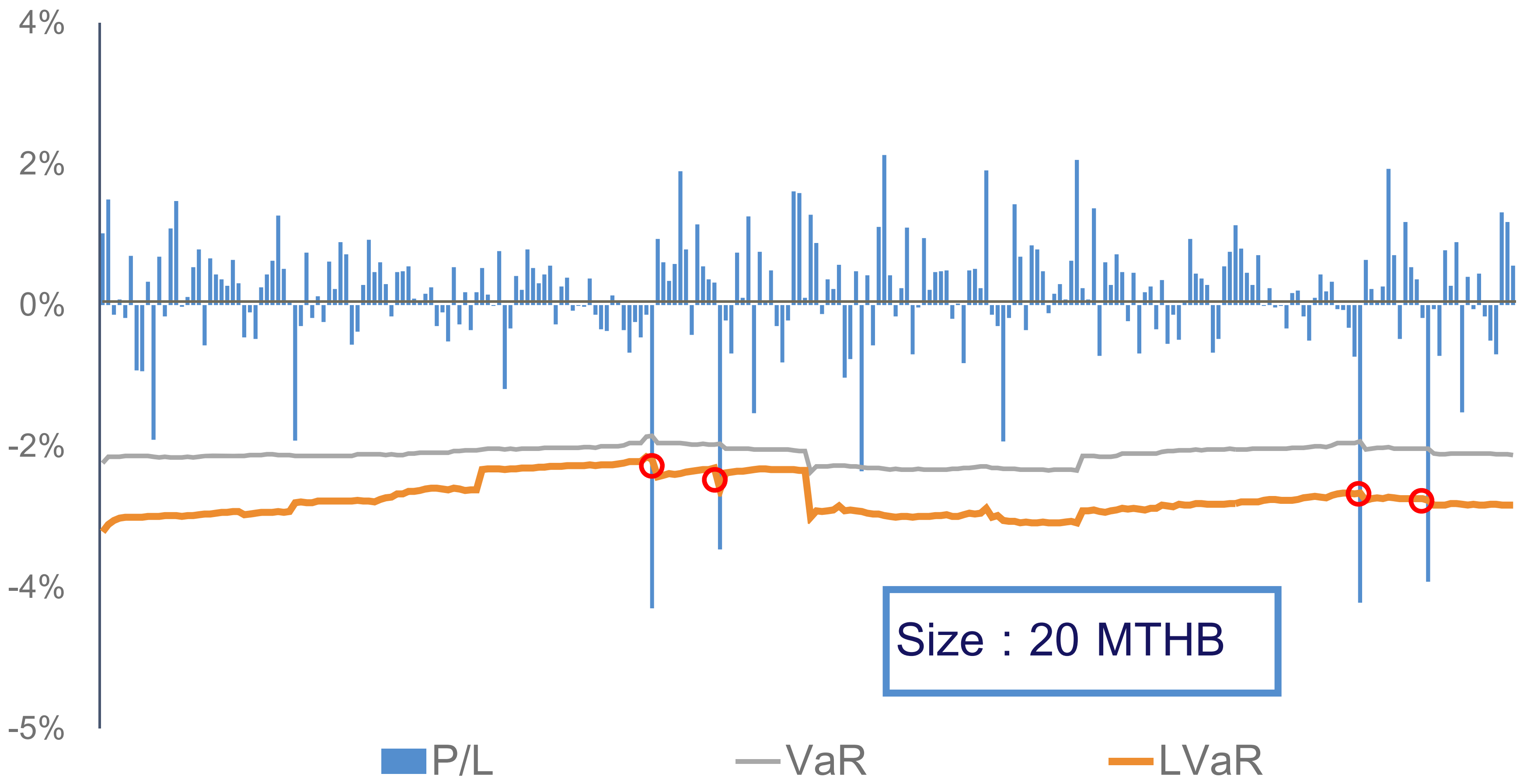
Low Liquid Portfolio



Suebsak (2017)

Backtesting for different levels of liquidity portfolios

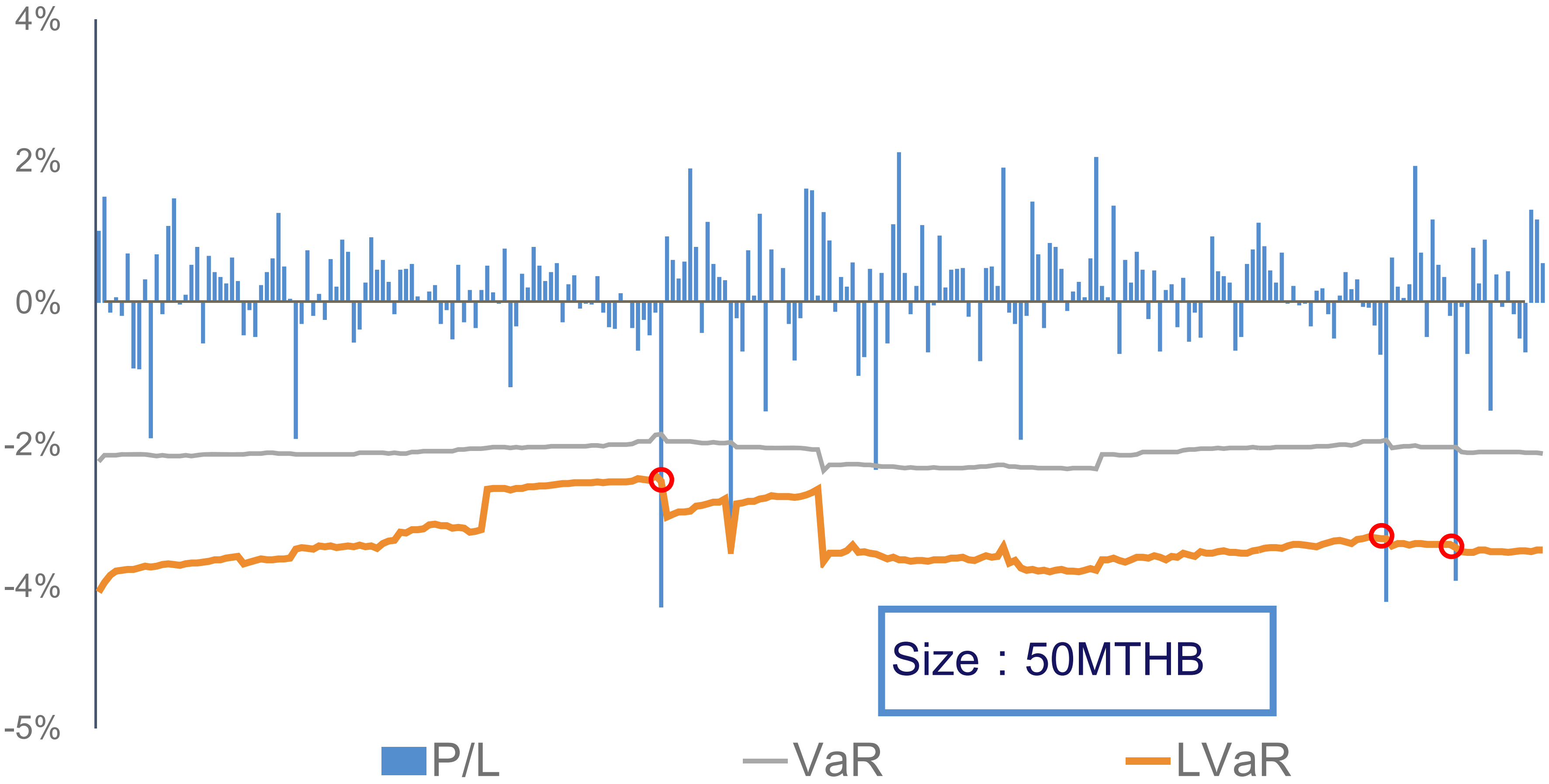
Low Liquid Portfolio



Suebsak (2017)

Backtesting for different levels of liquidity portfolios

Low Liquid Portfolio



Suebsak (2017)

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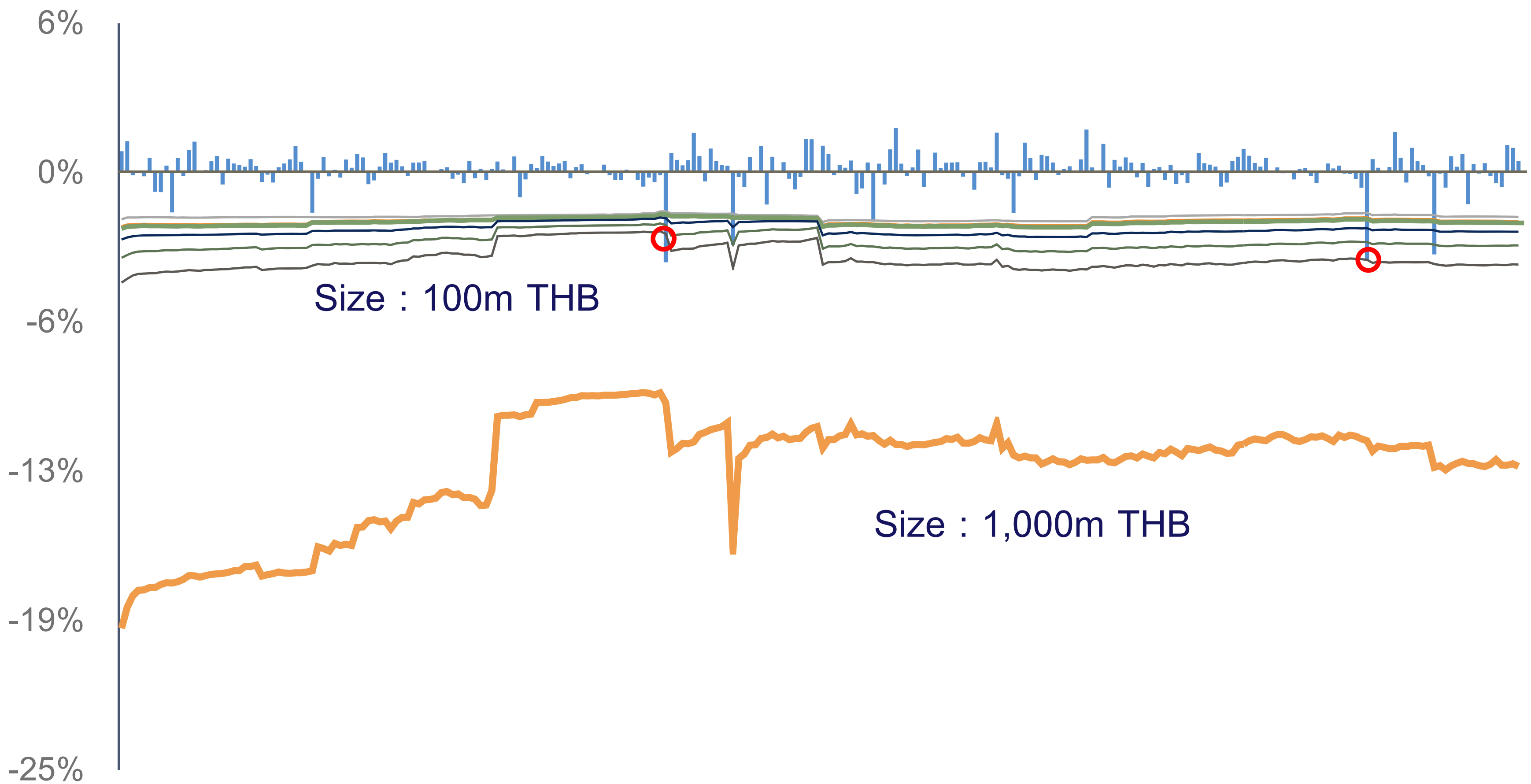

Suebsak (2017)

Summary of backtesting results

BIS Backtesting framework

	HIGH LIQUID PORTFOLIO						LOW LIQUID PORTFOLIO					
Observation	250						250					
Size (MTHB)	500	1,000	1,500	2,000	10,000	50,000	2.5	5	20	50	100	1,000
	Number of Exceptions						Number of Exceptions					
VaR	0	0	0	0	0	0	5	5	5	5	5	5
LVaR	0	0	0	0	0	0	4	4	4	3	2	0

Suebsak (2017)s

End of Presentation