การประชุมวิชาการ

“*ศาสตราจารย์สังเวียน อินทรวิชัย* ด้านตลาดการเงินไทย”

ครั้งที่ 24 ประจำปี 2559

การบรรยายเรื่อง

“Multivariate GARCH-M Model for Tactical Asset Re-allocation”

ผู้บรรยาย ดร.พูนศักดิ์ โล่ห์สุนทร

วันศุกร์ที่ 25 พฤศจิกายน 2559 เวลา 15:00 – 16:30 น.

ห้อง 207 คณะพาณิชยศาสตร์และการบัญชี

มหาวิทยาลัยธรรมศาสตร์ ท่าพระจันทร์

Multivariate GARCH for Tactical Asset Allocation

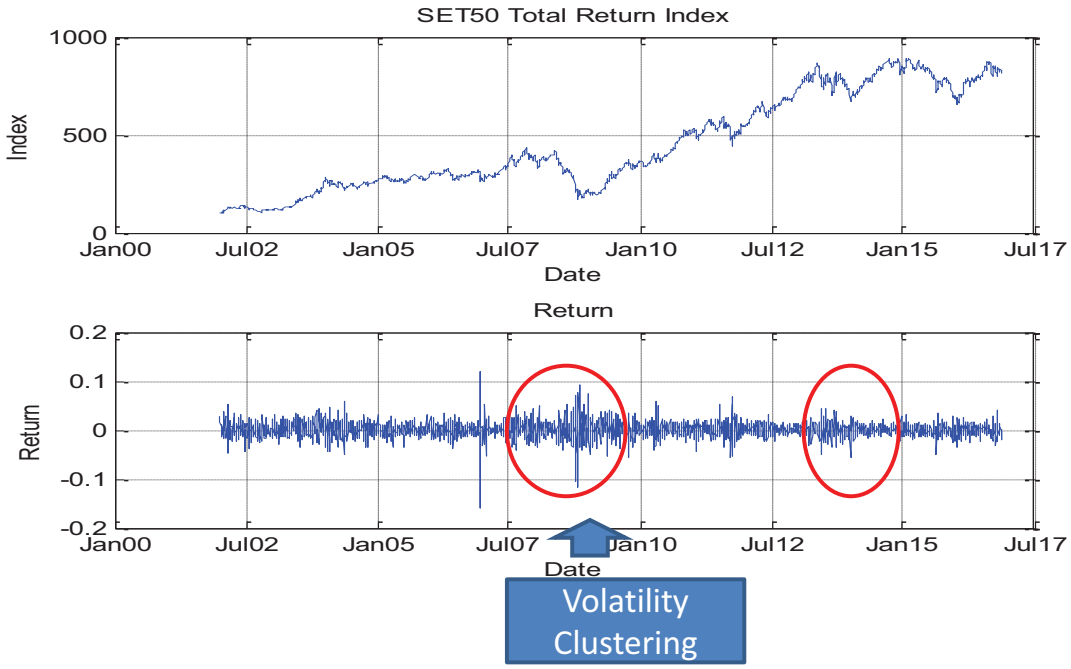
Dr. Poonsuk Lohsoonthorn

SCBAM

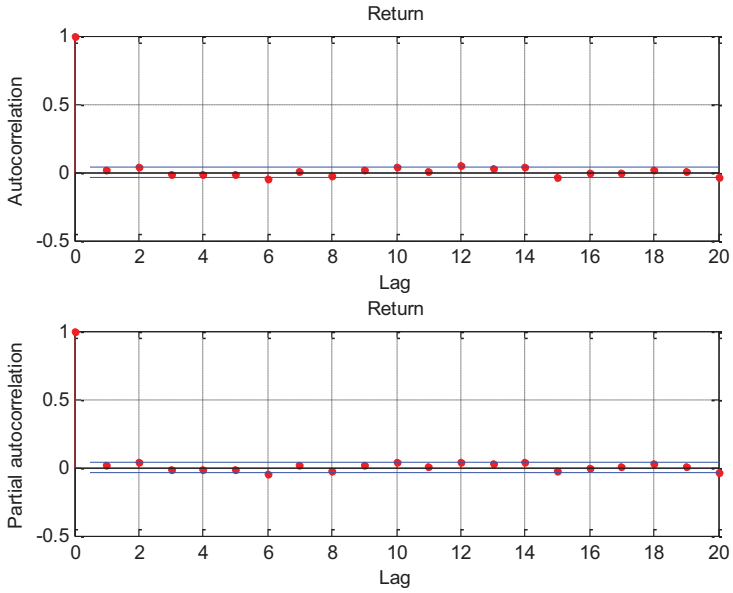
Outline

- Stylized Fact of Financial Data
- Review GARCH Model
  - Risk Control Index
- Multivariate GARCH
- Application
  - Minimum Volatility Portfolio
  - Risk Parity Portfolio
  - Max Sharpe Ratio Portfolio
- Conclusion

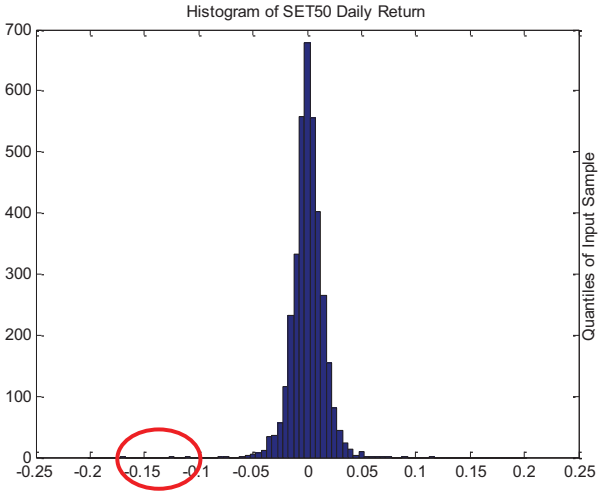
Stylized Facts of Daily Asset Return: SET50 Daily Data from 2002-2016

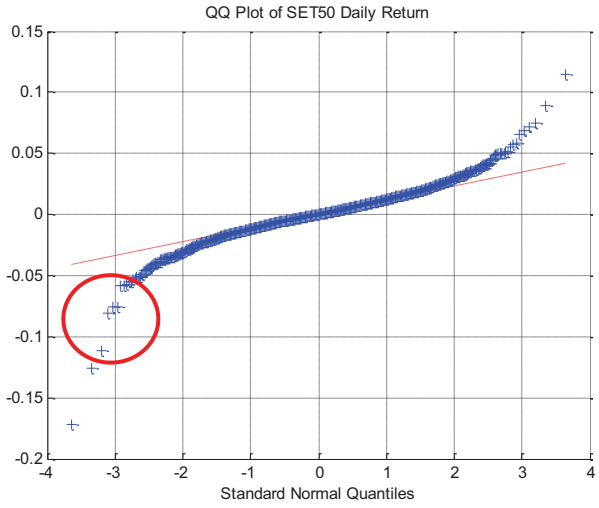


Stylized Facts of Daily Asset Return: SET50 Daily Data from 2002-2016



Stylized Facts of Daily Asset Return: SET50 Daily Data from 2002-2016

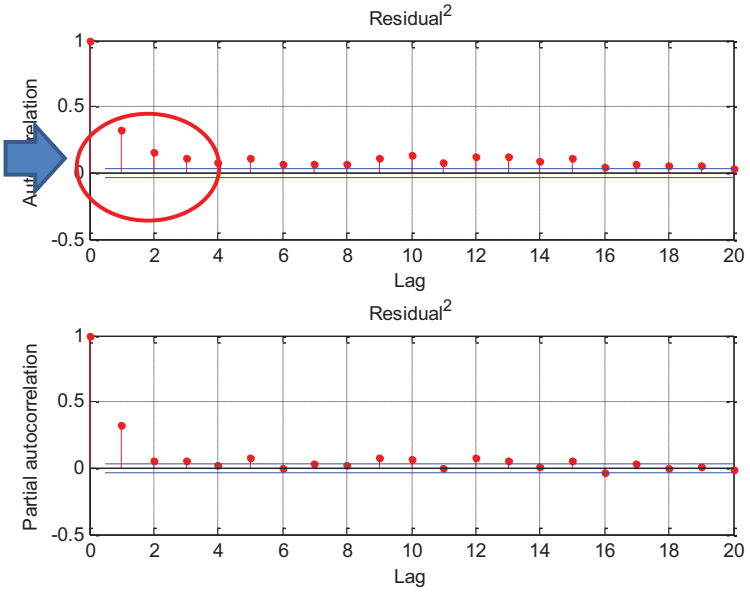




Heavy Tail, Extreme Value
Highly Non Normal

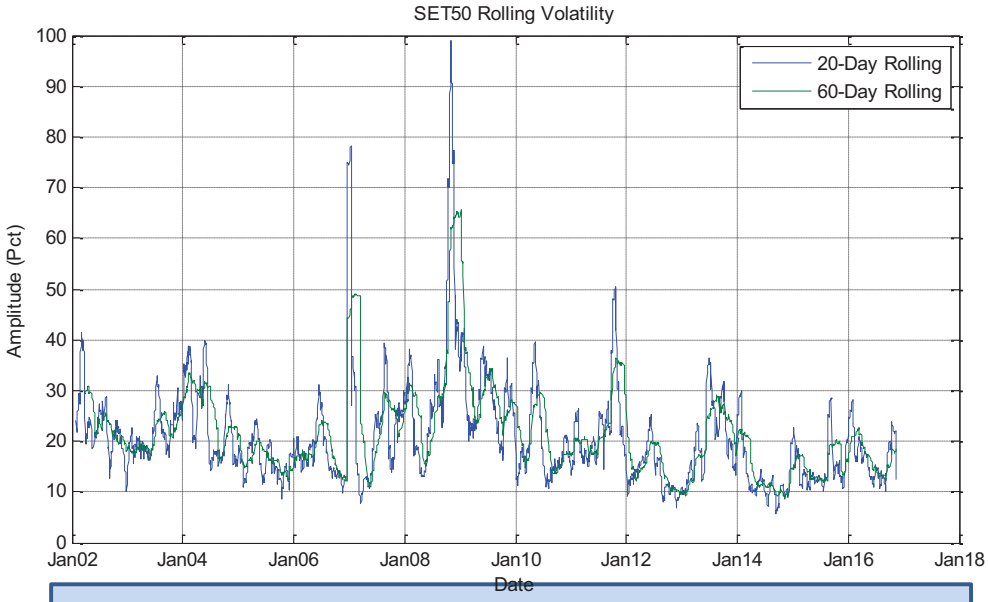
Stylized Facts of Volatility: SET50 Daily Data from 2002-2016

Dependence in Volatility



ARCH Test:
[h pValue stat cValue] = [1 0 380.59 3.84]
LBQ Test
[h pValue stat cValue] = [1 0 979.73 31.41]

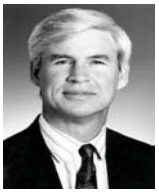
Stylized Facts of Volatility: SET50 Daily Data from 2002-2016



Volatility is time varying, clustering, and heteroscedasticity

Review GARCH Model

- ARCH (Autoregressive Conditional Heteroskedasticity) proposed by Robert Engle in 1982



- GARCH (Generalized ARCH) proposed by Bollerslev in 1986

- Engle and Clive Granger receive d Nobel Price in 2003

  "for methods of analyzing economic time series with time-varying volatility (ARCH)"

Review GARCH Model

- GARCH(P,Q) Process:

$$u_t = \sigma_t \varepsilon_t, \varepsilon_t \sim N(0,1)$$

$$\sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i u_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma_{t-j}^2, a_i, b_j \geq 0$$

Variance is predicted to be weight average of Long run average variance, current variance, and the news (today's square return)

Review GARCH Model

- GARCH Parameter Estimation: Maximum Likelihood Estimator (MLE)
- If error are not normal, the estimator is Quasi Maximum Likelihood Estimator (QMLE)
- MATLAB: GARCHFIT to estimate ARMAX/GARCH model parameters

Review GARCH Model

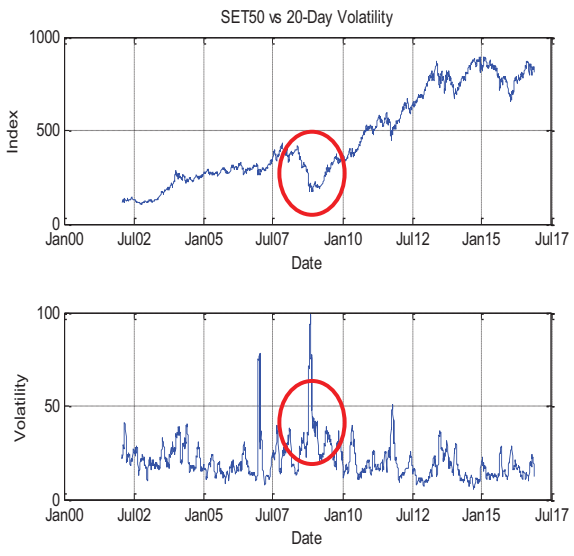
Bollerslev (2008)

- "Glossary to ARCH (GARCH)" encyclopedic reference guide to the long list of ARCH acronyms
- Identified over 150 model extension of ARCH
- Example: GJR-GARCH, AGARCH, EGARCH, FIGARCH, IGARCH, TARCH

Portfolio Strategy: Risk Control Index

Why Risk Control Works?

- Negative relationship between volatility and asset return
- Idea: dynamic asset allocation between underlying index and cash
- If the volatility is above the target, money is shifted to cash
- If the volatility is below the target, leverage is taken.



Portfolio Strategy: Risk Control Index

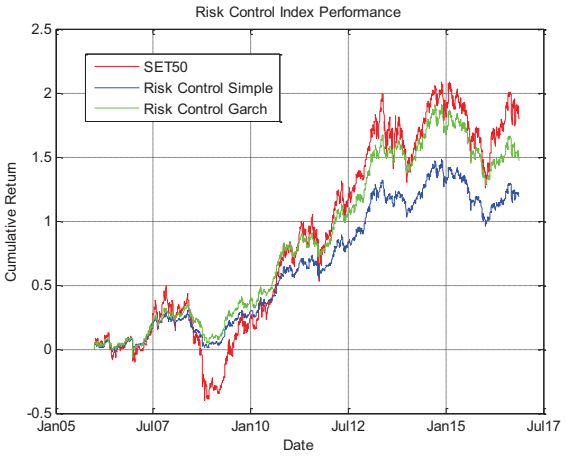
- Target specific level of Risk
- Varying weight of underlying asset and cash
  - Lower realized volatility than target, Increase weight of underlying asset
  - Higher realized volatility than target, decrease weight of underlying asset
- Realized Volatility calculate as max(Short term Volatility , Long Term Volatility)
- Weight of underlying Asset = Min(Max leverage, Target Volatility/Realized Volatility)

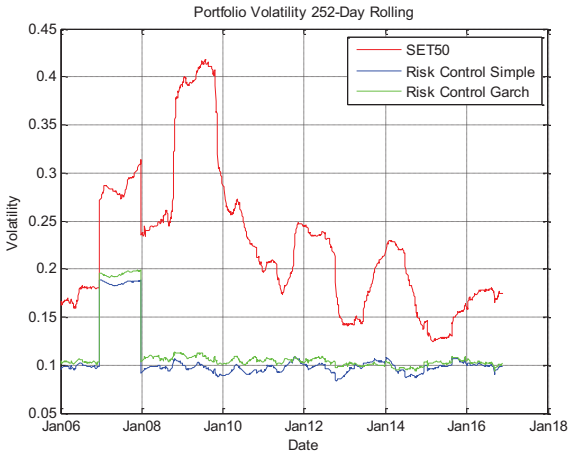
Portfolio Strategy: Risk Control Index

Compare realized volatility estimate by simple estimate and GARCH estimate

- Period: 2006 – 2016
- Underlying Asset: SET50
- Cash: ZRR6M Index
- Target Risk 10%
- Max leverage 1.5x
- Short term volatility 20 Day, Long term volatility 60 Day

Portfolio Strategy: Risk Control Index





Performance	SET50	Simple	Garch
Avg Return	12.5%	8.0%	9.2%
STD	23.6%	10.8%	11.5%
Sharpe Ratio	53.0%	73.6%	80.3%
MaxDrawDown	60.0%	22.4%	23.6%

Both method show similar result, Max drawdown for Risk Control Index is less than original index

Multivariate Problem

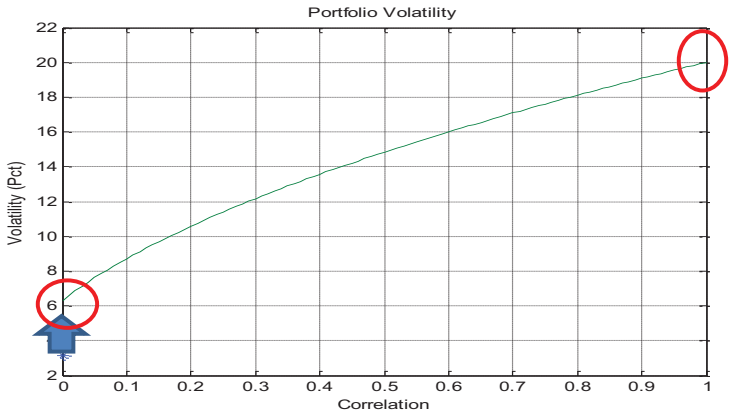
- Asset Allocation and Risk Management problems require to estimate large covariance matrices
- Extension from univariate GARCH to multivariate GARCH is not simple.
  - Estimation should be flexible
  - It should allow for covariance spillovers
  - Conditional covariance matrix should be positive definite
- 2 approaches: model covariance or model volatility and correlation

Correlation Effect

Correlation between asset as key driver for portfolio volatility

Consider portfolio of equally weight with 10 uncorrelated assets with volatility of 10%, portfolio volatility is 3.16%

- When volatility jump from 10% to 20% portfolio volatility move to 6.32%
- When pairwise correlation increase from 0 to 1, portfolio volatility change from 3.16 to 20%



Multivariate GARCH Model

- Extension from a univariate GARCH model to multivariate GARCH Model

$$\mathbf{u}_t = \mathbf{H}_t^{1/2}\boldsymbol{\varepsilon}_t, \boldsymbol{\varepsilon}_t \sim N(0,\mathbf{I})$$

$$vech(\mathbf{H}_t) = C + \sum_{i=1}^{p} A_i vech(\mathbf{u}_{t-i}\mathbf{u}_{t-i}^T) + \sum_{j=1}^{q} B_j vech(\mathbf{H}_{t-j})$$

Covariance matrix should be positive definite

Too many parameter to estimate
Curse of dimensionality ~o(n^4)

Multivariate GARCH-BEKK Model

- BEKK (Baba, Engle, Kraft and Kroner) GARCH
- Extension from a univariate GARCH model to multivariate GARCH Model

$$\mathbf{u}_t = \mathbf{H}_t^{1/2}\boldsymbol{\varepsilon}_t, \boldsymbol{\varepsilon}_t \sim N(0,\mathbf{I})$$

$$\mathbf{H}_t = CC^T + \sum_{i=1}^{p} A_i \mathbf{u}_{t-i}\mathbf{u}_{t-i}^T A_i^T + \sum_{j=1}^{q} B_j \mathbf{H}_{t-j} B_j^T$$

Multivariate GARCH-DCC Model

- Conditional covariance can expressed as follows:

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t,$$

- 2 Step approach
  - Estimate GARCH for each conditional volatility
  - Then the standardized residual can be estimated as follows:

$$\mathbf{v}_t = \mathbf{D}_t^{-1}\mathbf{u}_t, \quad \mathbf{v}_t \sim N(0,\mathbf{R}_t)$$

$$\overline{R} = \frac{1}{T}\sum_{t=1}^{T} \mathbf{v}_t \mathbf{v}_t^T$$

$$\mathbf{Q}_t = \overline{R} + \sum_{i=1}^{p} a_i \mathbf{u}_{t-i}\mathbf{u}_{t-i}^T + \sum_{j=1}^{q} b_j \mathbf{Q}_{t-j}$$

Exponentially Weighted Moving Average (EWMA)

- Riskmetrics
- Lambda: 0.94 for daily data, 0.97 for monthly data

$$\mathbf{u}_t = \mathbf{H}_t^{1/2}\boldsymbol{\varepsilon}_t, \boldsymbol{\varepsilon}_t \sim N(0,\mathbf{I})$$
$$\mathbf{H}_t = \lambda\mathbf{H}_{t-1} + (1-\lambda)\mathbf{u}_{t-1}\mathbf{u}_{t-1}^T, \quad 0 < \lambda < 1$$

Multivariate GARCH and Asset Allocation

Study the following Portfolio Strategy

- Minimum Volatility Portfolio
- Risk Parity Portfolio
- Max Sharpe Ratio Portfolio

With respect to different covariance estimator

- Simple Estimate
- VAR with Multivariate GARCH
- VAR with EWMA

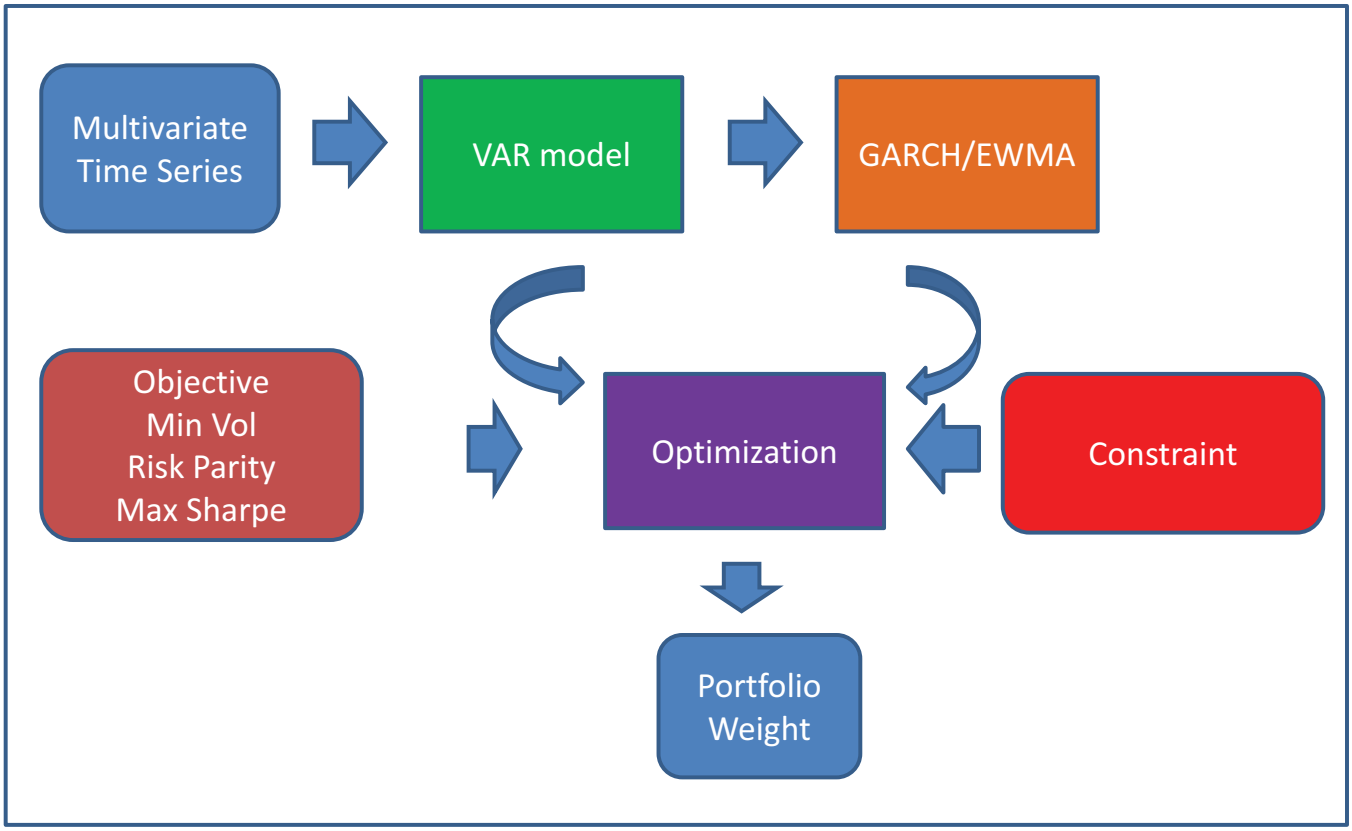
Vector Autoregressive Model (VAR)

- Extension from univariate AR model to multivariate time series model
- Vector Autoregressive (VAR)

$$\mathbf{y}_t = \mathbf{c} + \Phi_1 \mathbf{y}_{t-1} + \Phi_2 \mathbf{y}_{t-2} + \ldots + \Phi_p \mathbf{y}_{t-p} + \mathbf{u}_t, \textit{where}$$

$$\Phi_i \textit{ is } k \times k \textit{ matrix}$$

Methodology: VAR+GARCH/EWMA



Portfolio Strategy: Minimum Volatility

Rationale

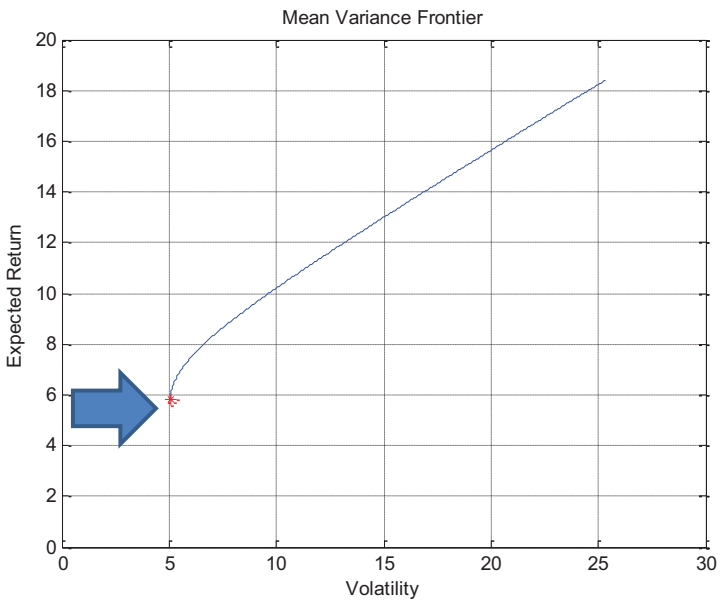
- Black, Jensen, Scholes (1972) portfolio long low beta stocks and short high beta stocks generated positive return
- Fund manager chase higher risk stocks to achieve returns, bidding up the prices in process, on the other hand less demand in lower risk stock increase upside potential
- Behavior biases: investor looking to make big bet in riskier, more expensive stock
- Bear market, more volatile, beta more disperse, low beta stock forms a buffer against falling market, Bull market, beta spread is tight, high beta stock outperformance is limited

Portfolio Strategy: Minimum Volatility

- Methodology: construct minimum volatility portfolio by

$$\min_{w} w^T \Omega w$$

Minimum Volatility Portfolio

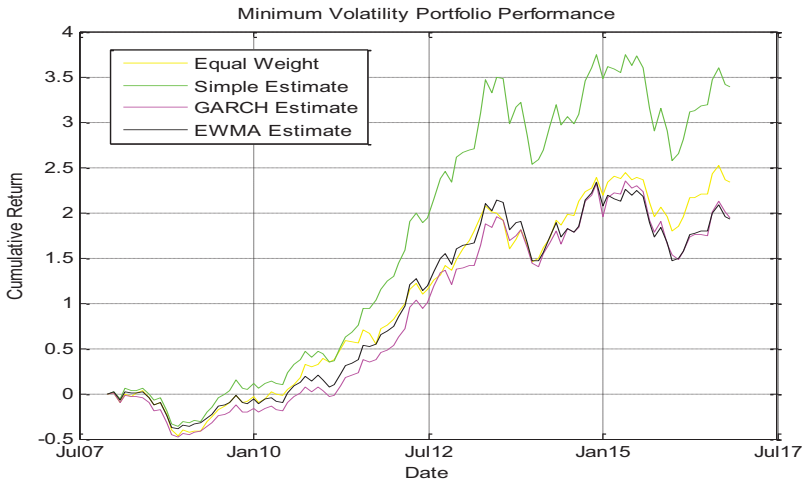


Portfolio Strategy: Minimum Volatility

Comparison among minimum volatility using different Covariance Estimator

- Period: Monthly Data from 2008 -2016
- Stock Universe: PTT SCC SCB ADVANC BDMS
- No leverage
- Monthly Rebalance
- Minimum volatility based on optimization
- Comparison among 3 strategy: Simple Estimate, VAR with GARCH, VAR with EWMA

Portfolio Strategy: Minimum Volatility



Performance	Equal	Simple	GARCH	EWMA
Avg Return	15.9%	18.9%	14.4%	14.1%
STD	20.8%	21.3%	21.5%	20.3%
Sharpe Ratio	76.1%	88.6%	67.2%	69.2%
MaxDrawDown	47.7%	40.5%	48.2%	40.0%

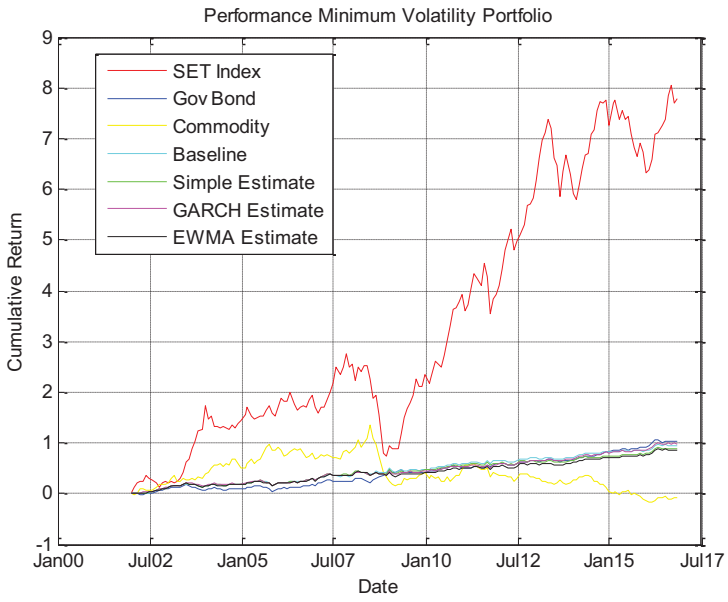
EWMA show lower portfolio volatility

Portfolio Strategy: Risk Parity

Comparison among minimum volatility using different Covariance Estimator

- Period: Monthly Data from 2002 -2016
- Asset: SET, TBMA Gov. Bond, Commodity
- Cash: ZRR6M
- Method: Minimum Volatility
- Monthly Rebalance
- Comparison among 3 strategy: Simple Estimate, VAR with GARCH, VAR with EWMA

Portfolio Strategy: Minimum Volatility



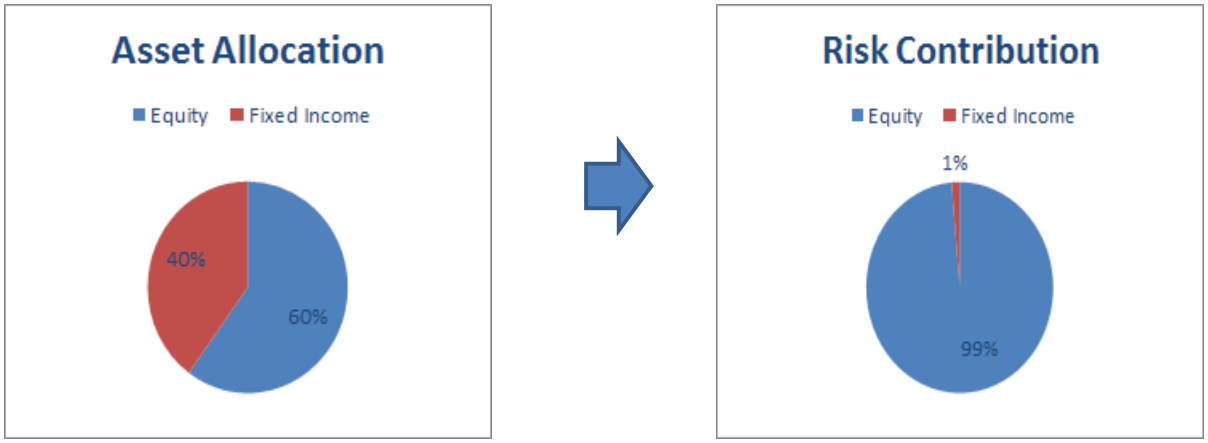
Minimum Volatility Portfolio

Performance	Baseline	Simple	GARCH	EWMA
Avg Return	4.6%	4.4%	4.8%	4.3%
STD	4.9%	5.0%	5.0%	5.0%
Sharpe Ratio	0.95	0.89	0.96	0.86
MaxDrawDown	7.8%	8.6%	9.5%	8.6%

Portfolio Strategy: Risk Parity

Rationale

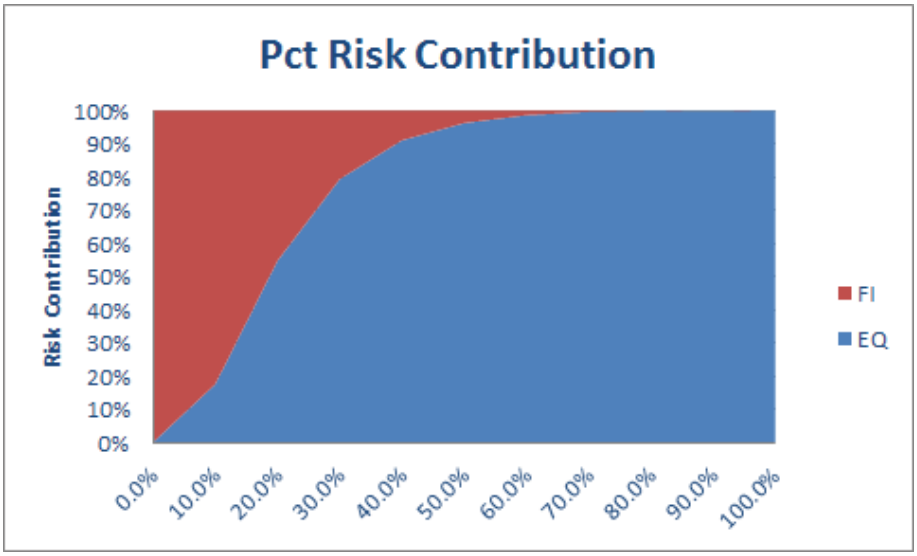
- Traditional balance fund 60/40 do not offer investors true diversification



- 60/40 Portfolio have too much equity risk
- Portfolio performance mainly depend on Equity return

Portfolio Strategy: Risk Parity

- Observe that when equity weight move from 10% to 20%, equity risk contribution move from 18% to 55%



Portfolio Strategy: Risk Parity

- Pioneer concept: Ray Dalio, Bridgewater Associates, The All Weather Strategy
  - "What kind of investment portfolio would you hold that would perform well across all environments, be it a devaluation or something completely different?'

MARKET EXPECTATIONS	Growth	Inflation
Rising	25% OF RISK Equities Commodities Corporate Credit EM Credit	25% OF RISK IL Bonds Commodities EM Credit
Falling	25% OF RISK Nominal Bonds IL Bonds	25% OF RISK Equities Nominal Bonds

Portfolio Strategy: Risk Parity

- Portfolio volatility

$$\sigma(w) = \sqrt{w^T \Omega w}$$

- Marginal Risk Contribution

$$\sigma_i(w) = \frac{w_i(\Omega w)_i}{\sqrt{w^T \Omega w}}$$

- Equal Risk Contribution

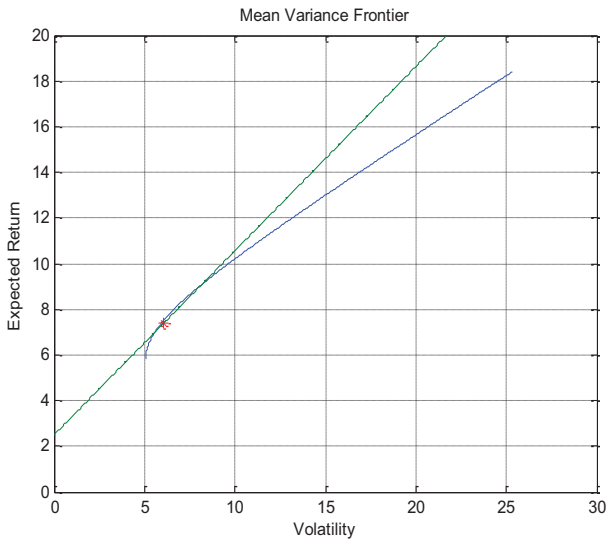
$$\sigma_i(w) = \frac{\sigma(w)}{N}, \Rightarrow w_i = \frac{(\sigma(w))^2}{N(\Omega w)_i}$$

- Risk Parity

$$\min_w \sum_i \left(w_i - \frac{(\sigma(w))^2}{N(\Omega w)_i} \right)^2$$

Portfolio Strategy: Risk Parity

- Risk Parity focus on risk diversification while minimum volatility focus on risk reduction on portfolio level which is more sensitive to volatility and correlation input
- Unlevered risk parity portfolio may attractive in term of diversification but expected return may be too low.
- Risk parity portfolio can be scaled to match an investor desired expected return.



Portfolio Strategy: Risk Parity

Cliff Asness, AQR “Risk Parity; Why We Lever”

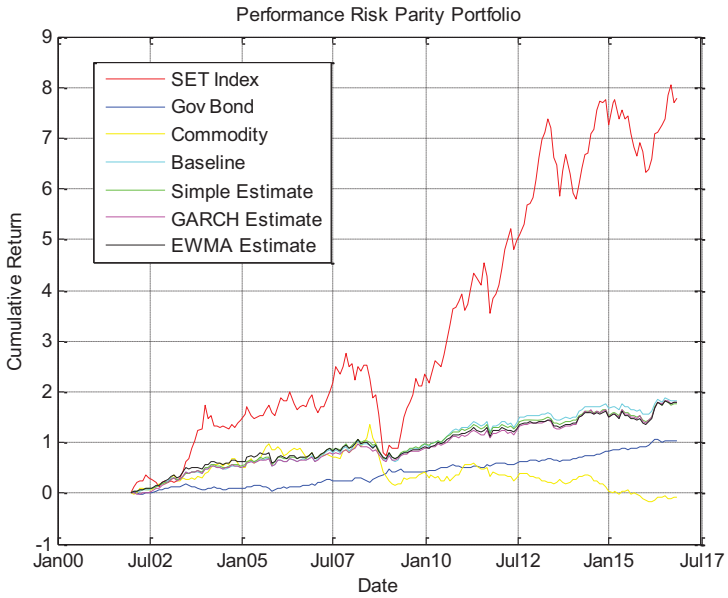
- Asset allocation must be balanced by risk, not by dollars.
- If, and this will most commonly be the case, after your best efforts at balancing risk across the asset classes, your expected return is too low, leverage should be applied to this portfolio rather than changing the allocation toward higher-return asset classes.

Portfolio Strategy: Risk Parity

Comparison among risk parity strategy using different Covariance Estimator

- Period: Monthly Data from 2002 -2016
- Asset: SET, TBMA Gov. Bond, Commodity
- Cash: ZRR6M
- Method: Risk Parity Portfolio with target volatility 10%
- Monthly Rebalance
- Comparison among 3 strategy: Simple Estimate, VAR with GARCH, VAR with EWMA

Portfolio Strategy: Risk Parity



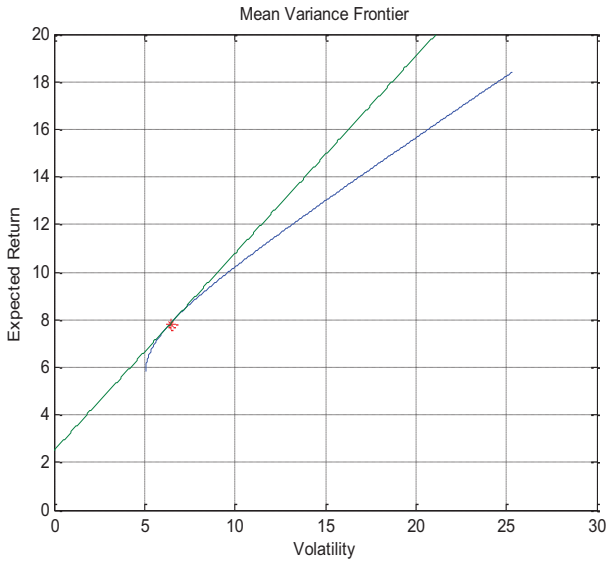
Risk Parity Portfolio (Target Volatility 10%)

Performance	Baseline	Simple	GARCH	EWMA
Avg Return	7.5%	7.3%	7.3%	7.4%
STD	9.1%	8.9%	9.0%	9.6%
Sharpe Ratio	0.82	0.82	0.81	0.77
MaxDrawDown	19.8%	19.2%	16.8%	19.0%

GARCH provide similar Sharpe Ratio as baseline model but its max drawdown is lower

Portfolio Strategy: Max Sharpe Ratio

- Maximum Sharpe Ratio portfolio is portfolio that tangent to CAL
- Adjust portfolio volatility along the CAL.
- TAA based on one step prediction of the expected return based on VAR and the estimate covariance with GARCH/EWMA
- Benchmark portfolio is based on long term expected return and covariance

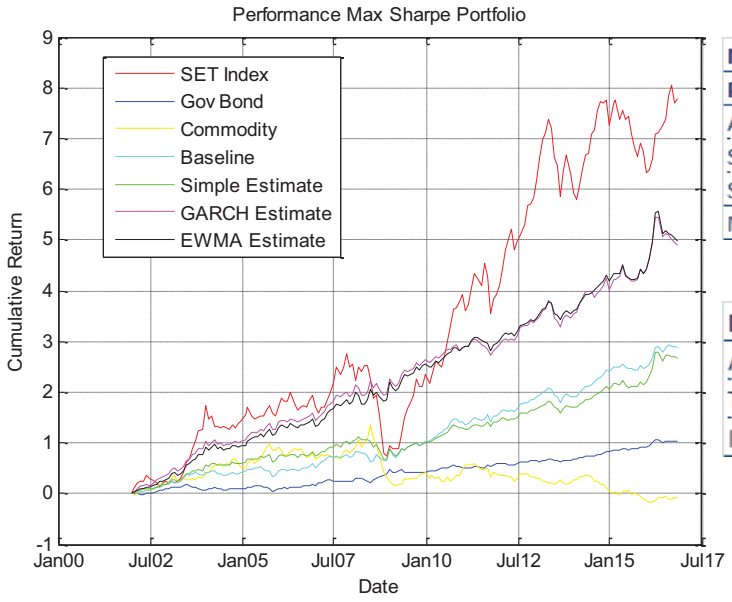


Portfolio Strategy: Max Sharpe Ratio

Comparison among maximum Sharpe ratio strategy using different Covariance Estimator

- Period Monthly Data from 2002 -2016
- Asset: SET, TBMA Gov. Bond, Commodity
- Cash: ZRR6M
- Method: Max Sharpe Ratio portfolio with target volatility 10%
- Monthly Rebalance
- Comparison among 3 strategy: Simple Estimate, VAR with GARCH, VAR with EWMA

Portfolio Strategy: Max Sharpe Ratio



Max Sharpe Ratio Portfolio (Target Volatility 10%)				
Performance	Baseline	Simple	GARCH	EWMA
Avg Return	9.6%	9.2%	12.5%	12.6%
STD	9.0%	9.0%	9.6%	9.5%
Sharpe Ratio	1.06	1.03	1.30	1.32
MaxDrawDown	14.1%	20.3%	11.0%	8.8%

Performance	Simple	GARCH	EWMA
Alpha	-0.4%	2.9%	3.0%
TE	4.8%	7.8%	7.5%
IR	-0.08	0.37	0.40

VAR with GARCH/EWMA provide positive alpha with respect to baseline model, Both model have slightly higher volatility but lower max drawdown

Conclusion

- Several estimators yield similar performance for minimum volatility portfolio
- VAR with Multivariate GARCH can improve Risk Parity portfolio by lower portfolio max drawdown
- VAR with Multivariate GARCH/EWMA provide can use to improve portfolio construction for Tactical Asset Allocation
- While Multivariate GARCH involve a lot of parameter estimations, EWMA is easier to estimate .