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Macroprudential Policy in a Bubble-Creation Economy

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Abstract

This paper analyzes macroprudential policy in the form of loan-to-value (LTV) restriction in a bubble-creation economy of Martin and Ventura (forthcoming). We find that implementation of LTV policy may generate multiple equilibria. Moreover, its effectiveness in terms of investment and size of bubbles depends on the degree of financial friction. In high-capital steady state, low (high) financial friction implies that bubbles originally crowd out (in) investment, so that implementation of LTV policy causes bubbles to decrease (remain unchanged) and enhances (reduces) investment. However, in low-capital equilibrium, the policy has ambiguous effects. LTV policy may help to lower the possibility of sunspot equilibria in two aspects: (1) by destabilizing the low-capital steady state and (2) by confining the set of consistent market sentiments in the presence of high financial friction.

JEL Classification: E44, F41, G12

Keywords: Rational bubbles, Bubble creation, Macroprudential policy, Loan-to-value ratio, Overlapping generations model, Financial friction.

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1 Introduction

The best lessons are learned from real pain. Because of the long line of financial crises, economists are now well aware of how asset price boom and bust can severely affect the real economy. Since the Global Financial Crisis of 2008, there have been considerable discussions about how macroprudential policy may complement monetary policy as a stabilization tool. Countries such as Thailand have been implementing macroprudential tools (as a counter-cyclical measure) with the aim of managing credit booms, particularly in the real-estate sector. See, for example, IMF (2013) on key aspects of macroprudential policy.

Macroprudential policy and monetary policy are often studied in a dynamic stochastic general equilibrium framework, which by design has ruled out asset price bubbles. Gali (2014) has studied the relationship between monetary policy and bubbles in the overlapping-generations environment, which allows for the existence of asset price bubbles, and called into question the conventional wisdom of "leaning against the wind" policy in the face of bubble fluctuations. This dilemma would also call for a more careful theoretical investigation of macroprudential measures in the bubbly environment.

Along the same lines as Gali (2014), the present paper uses the overlapping-generations framework to revisit the use of the loan-to-value (LTV) ratio as a macroprudential tool in the theoretical model of rational bubbles. Specifically, we employ Martin and Ventura's (2015) model, which is a modified version of Martin and Ventura (2012). The present paper also analyzes various effects of the LTV ratio on investment, bubbles, and economic fluctuation. To the best of the authors' knowledge, this is the first paper analyzing the effects of macroprudential policy in a *bubbly* environment.

The literature on rational bubbles has advanced greatly over the past three decades. Tirole (1985) and Weil (1987), and Santos and Woodford (1997) have shown that bubbles can emerge as stores of value to help solve the shortage of financial instruments in the overlapping-generations model. In a dynamically inefficient economy, bubbles absorb savings out of inefficient investment to raise the rate of return. Thus, bubbles are favorable. However, Tirole's results contradict empirical evidence which shows that most economies with bubble episodes are in fact dynamically efficient and that bubble booms are often accompanied by investment booms (see, e.g., Abel, Mankiw, and Zeckhauser (1989)).

Following studies of the balance-sheet effect of credit constraints, such as Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), many studies have examined how bubbles can exist in a dynamically efficient economy and how bubbles crowd in investment (see, e.g., Caballero and Krishnamurthy (2006), Caballero, Farhi and Hammour (2006), Kocherlakota (2009), Farhi and Tirole (2012), Martin and Ventura (2011), Ventura (2012), and Hirano and Yanagawa (2013)). Although different models have different details, they share the same key element: financial imperfection. In the dynamically efficient economy, limited pledgeability generates a credit constraint, which suppresses the demand for capital. As a result, the rate of interest falls below the growth rate of the economy. This creates room for bubbles. If the economy is initially credit constrained, bubbles can act as collateral to relax the credit constraint and raise the demand for capital. Thus, bubbles can crowd in investment.

Recently, Martin and Ventura (2012, 2015) have made important progress in rational bubble modeling. They introduce so-called bubble creation. That is, in every period, new bubbles can be created out of nothing as long as they are consistent with the prevailing belief of agents. Throughout the present paper, this bubble-creation belief is also referred to as market sentiment. Bubble creation helps to relax the bubble no-arbitrage condition and notably enlarge the bubbly equilibrium feasible set. Martin and Ventura (2012, 2015) also show that there is an optimal bubble creation that provides maximum investment and that it can be achieved by fiscal intervention to transfer wealth across time.

Using this model as the benchmark, we add on the LTV ratio to the bubbly collateral. In simple terms, it is regulated so that only a fraction of bubbles can be pledged. The LTV ratio works differently from the fiscal intervention policy discussed by Martin and Ventura (forthcoming) because the LTV ratio is distortionary. Changing the LTV ratio also changes the optimal bubble-creation level.

Unexpectedly, implementation of the LTV ratio generates multiplicity and hence additional short-run fluctuation. In particular, the LTV ratio allows only a part of the bubbles to be pledgeable and helps to relax the credit constraint, while the remaining part becomes the burden competing with investment over the loan. Consequently, these two opposing effects make the capital demand curve backward-bending and hence cause short-run indeterminacy in the capital market.

In the long run, through analytical and numerical investigation, we find that the model has at most two steady states and that the effects of the LTV ratio on each steady state differ depending on the degree of financial friction. First, we consider the effects on the high-capital steady state. When the financial friction is low, lowering the LTV ratio raises investment and reduces bubbles of the high-capital steady state. When the financial friction is high, investment declines, but bubbles remain unaffected. The LTV ratio reduces the benefit of bubbles as collateral and hence becomes a disincentive for bubble holding. Decrease in bubbles would increase capital if the financial friction is not severe, which means that the fundamental collateral is sufficient. However, when the financial friction is high, the rate of interest is already at the minimum rate of time preference, and hence bubbles cannot decrease. As a result, lowering the LTV ratio only results in reduction of investment.

The abovementioned effects are logically expected. Basically, given low financial friction, bubbles are against investment, and therefore implementing the LTV ratio is effective. Otherwise, the LTV ratio damages investment only. However, effects of the LTV ratio on investment and bubbles are more complicated and ambiguous in the low-capital steady state. We find that lowering the LTV ratio may deliver the most undesirable outcome, namely, investment decreases and bubbles increase. An inability to know in which steady state the economy will be located means that implementation of LTV ratio policy is challenging.

Although LTV ratio effects on the low-capital steady state are ambiguous, we find that the low-capital steady state tends to lose its stability as the LTV ratio decreases.[1] This implies that the LTV ratio may help to eliminate the fluctuation originating from sunspot equilibria where the economy switches between low-capital and high-capital equilibrium for a given market sentiment.

In contrast, fluctuation caused by changes in market sentiments cannot be entirely eliminated. However, we can confine the set of market sentiments which is consistent with rational expectations equilibrium and hence reduce this type of fluctuation. As the LTV ratio decreases, the set of consistent market sentiments expands if the financial friction is sufficiently low, but the set shrinks if the financial friction is sufficiently high. Logically, low financial friction leads to high

[1] The exception is when the financial friction is low and the LTV ratio is already at a sufficiently low level. In this case, the low-capital steady state returns to stability.

demand for capital and thus high rate of interest. High market sentiment would raise the demand for capital further, with the rate of interest exceeding the growth rate of the economy, which violates the existence condition of rational bubbles. Lowering the LTV ratio would lower the demand for capital and provide more room for market sentiment. Conversely, when financial friction is high, little future income can be pledged. High market sentiment would imply large bubbles. Lowering the LTV ratio would tighten the credit limit even further to the extent that it would not be able to sustain such large bubbles and hence would provide less room for market sentiment.

The remainder of this paper is organized as follows. Section 2 outlines Martin and Ventura's (2015) model with the LTV ratio add-on. Section 3 analyzes the steady state. Section 4 investigates sunspot equilibria. Section 5 considers the effects of the LTV ratio on different types of fluctuation. Section 6 offers conclusions.

2 Setup: the basic bubble-creation model

As mentioned in the Introduction, we adapt Martin and Ventura's (2015) bubble-creation model. We consider a closed economy populated by overlapping generations who live for two periods. All markets are competitive. Each generation consists of two types of agents: workers (*savers*) and entrepreneurs (*lenders*). Each type has unit mass with no population growth, and there is no initial endowment. All agents are risk-neutral and can choose to consume at any point in their lifetime: $U(C_t) = C_t + \beta C_{t+1}$, where C_t is the consumption at period t and $\beta \in (0, 1]$ is the discount factor.

Young workers work for old entrepreneurs and receive a wage income. Then, they decide how much to lend to young entrepreneurs. When old, the entrepreneurs repay the debt to the workers, and then both entrepreneurs and workers consume all they have got. The young worker's maximization problem is as follows:

$$\max_{C_t, C_{t+1}, L_t} C_t + \beta C_{t+1}$$

subject to

$$C_t = W_t - L_t$$

$$C_{t+1} = R_{t+1} L_t$$

where W_t and L_t are, respectively, wage income and loan amount at period t, and R_{t+1} is the rate of interest at period $t+1$.

Young entrepreneurs borrow from young workers and decide how much to invest in next-period capital, how much to spend on bubbles, and how much to consume. Bubbles are goods with no intrinsic value, and people are willing to buy them simply because they believe that they can sell them to the next generation at the expected price. In other words, bubbles are another form of intergenerational wealth transfer that resembles a pay-as-you-go security system, but it is more fragile because it proceeds without a government guarantee. Young entrepreneurs demand bubbles because they have limited pledgeability, meaning that they cannot use their entire future production income as (fundamental) collateral in their debt contract. Bubbles in the form of assets can be additionally pledged as (bubbly) collateral to elevate the credit limit. The entrepreneur's credit constraint and the evolution of bubbles are respectively shown as follows:

$$\begin{aligned} R_{t+1} L_t &\leq \phi \left[F(K_{t+1}, N_{t+1}) - W_{t+1} N_{t+1} \right] + \lambda B_{t+1} \\ B_{t+1} &= R^B_{t+1} B_t + B^N_{t+1} \end{aligned}$$

where K_t, N_t, B_t, $B^N_t \geq 0$, and R^B_t are, respectively, capital, employed workers, bubbles, newly created bubbles, and growth rate of bubbles at period t; $F(.,.)$ is the production function of consumption goods; and λ is the LTV ratio. B^N_t is bubble creation, which is the distinguishing innovation of this model. Martin and Ventura's (2015) model lets agents take as given a sequence of B^N_t before making an economic decision, similar to taking price sequences. If there is a supporting equilibrium, the sequence of B^N_t is a consistent belief and indeed prevails in the equilibrium. In previous bubble literature, bubble evolution is tightly constrained by $B^N_t = 0$, which rules out many feasible equilibrium solutions.

The LTV ratio is a macroprudential policy, which is the main subject of this paper. It is the regulation from the central bank over the borrower to consider only a fraction of bubble assets as bubbly collateral. This fraction λ is officially announced and becomes common knowledge to all agents.[2]

When entrepreneurs are old they hire young workers and use their invested capital to produce consumption goods. We assume full depreciation of capital so that there is no capital left over after production. After repaying the debts and selling all bubbles, old entrepreneurs consume all the leftovers. The production function here is assumed to be of the standard Cobb–Douglas form with growing labor efficiency: $F(K_{t+1}, N_{t+1}) = A_{t+1}K_{t+1}^{\alpha}(\gamma^{t+1}N_{t+1})^{1-\alpha}$, where A_{t+1} is technological progress at period $t+1$, and γ is the growth rate of labor efficiency.[3] The entrepreneur's maximization problem is as follows:

$$
\begin{aligned}
&\max_{C_t, C_{t+1}, L_t} C_t + \beta C_{t+1} \\
&\text{subject to} \\
&C_t = L_t - K_{t+1} - B_t \\
&C_{t+1} = A_{t+1}K_{t+1}^{\alpha}(\gamma^{t+1}N_{t+1})^{1-\alpha} - W_{t+1}N_{t+1} + B_{t+1} - R_{t+1}L_t \\
&B_{t+1} = R_{t+1}^{B}B_t + B_{t+1}^{N} \\
&R_{t+1}L_t \leq \phi A_{t+1}K_{t+1}^{\alpha}(\gamma^{t+1}N_{t+1})^{1-\alpha} + \lambda B_{t+1}.
\end{aligned}
$$

From the young worker's maximization problem, standard risk-neutral preference implies that young workers would lend all their wage income if the expected rate of interest is more than the rate of time preference (β^{-1}), as summarized below:

$$
l_t \begin{cases} = w_t & ; R_{t+1} > \beta^{-1} \\ \in [0, w_t] & ; R_{t+1} = \beta^{-1} \end{cases} \tag{1}
$$

where variables per effective worker are presented as lower-case letters. For example, $l_t = \left(\gamma^{-t}N_t\right)^{-1} L_t$.

From the young entrepreneur's maximization problem, the optimal decision is as follows. First, entrepreneurs would hire workers until the marginal product

[2]The LTV ratio can be time-varying (λ_t). However, this will not change any result or insight of the model, so we assume it to be constant for simplicity.

[3]Realistically, A_{t+1} and B_t^N are drawn from some random processes. However, the stochastic model is unnecessary for the scope and purpose of this paper.

of labor is equal to the wage cost. Second, they would buy bubbles only if the bubbles grow at the rate of interest. Third, they would not consume anything while they are young if the marginal product of capital exceeds the rate of time preference. Finally, they would demand capital until the marginal product of capital is equal to the fund cost if the credit constraint is non-binding. However, if the credit constraint is binding, the capital demand is instead determined by the binding credit constraint. All these optimal conditions are presented respectively as follows:

$$w_{t+1} = (1-\alpha)A_{t+1}k_{t+1}^{\alpha} \tag{2}$$

$$R_{t+1}^{B} = R_{t+1} \tag{3}$$

$$\gamma k_{t+1} + b_t \begin{cases} = l_t & ; \alpha A_{t+1}k_{t+1}^{\alpha-1} > \beta^{-1} \\ \in [0, l_t] & ; \alpha A_{t+1}k_{t+1}^{\alpha-1} = \beta^{-1} \end{cases} \tag{4}$$

$$R_{t+1} = \begin{cases} \alpha A_{t+1}k_{t+1}^{\alpha-1} & ; \frac{\lambda b_{t+1}^{N}}{(1-\phi)\alpha A_{t+1}k_{t+1}^{\alpha}} \geq 1 + \frac{(1-\lambda)b_t}{\gamma(1-\phi)k_{t+1}} \\ \frac{\phi\alpha A_{t+1}k_{t+1}^{\alpha}+\lambda b_{t+1}^{N}}{k_{t+1}+\gamma^{-1}(1-\lambda)b_t} & ; \frac{\lambda b_{t+1}^{N}}{(1-\phi)\alpha A_{t+1}k_{t+1}^{\alpha}} < 1 + \frac{(1-\lambda)b_t}{\gamma(1-\phi)k_{t+1}} \end{cases} \tag{5}$$

To close the model, all markets must be cleared. There are four markets in total: labor, loanable fund, capital, and bubble. In the labor market, labor demand from (2) must be equal to labor supply, which is inelastically equal to 1. Similarly, in the loanable fund market, demand for loans from (4) must be equal to supply for loans from (1). These market-clearing conditions are presented respectively as follows:

$$N_t = 1 \tag{6}$$

$$\gamma k_{t+1} + b_t \quad \begin{cases} = w_t & ; R_{t+1} > \beta^{-1} \\ \in [0, w_t] & ; R_{t+1} = \beta^{-1} \end{cases} \tag{7}$$

Combining (1)–(7) with the market-clearing condition of the bubble market, the equilibrium system can be presented as follows:

$$k_{t+1} \begin{cases} = \gamma^{-1}(1-\alpha)A_t k_t^{\alpha} - \gamma^{-1}b_t & ; R_{t+1} > \beta^{-1} \\ \in \left[0, \gamma^{-1}(1-\alpha)A_t k_t^{\alpha} - \gamma^{-1}b_t\right] & ; R_{t+1} = \beta^{-1} \end{cases} \tag{8}$$

$$R_{t+1} = \min\left\{\alpha A_{t+1}k_{t+1}^{\alpha-1}, \frac{\phi\alpha A_{t+1}k_{t+1}^{\alpha} + \lambda b_{t+1}^{N}}{k_{t+1} + \gamma^{-1}(1-\lambda)b_t}\right\} \tag{9}$$

$$b_{t+1} = \gamma^{-1}R_{t+1}b_t + b_{t+1}^{N} \tag{10}$$

where k_0 and b_0 are given as initial values.

Basically, the above equilibrium system illustrates the other two market-clearing conditions. In the capital market, supply for capital from (8) must be equal to demand for capital from (9). Finally, (10), which is the law of motion equation for bubbles, implies that all bubbles available in the market must be purchased in every period.

It is useful to understand visually how this system works. Given today's capital and bubbles, (8) can be drawn as the kinked supply as shown in Figure 1: Workers supply all their wage income given as long as the rate of interest is above the rate of time preference; otherwise they consume part of their wage income, and lend the remainder only at the rate of time preference.

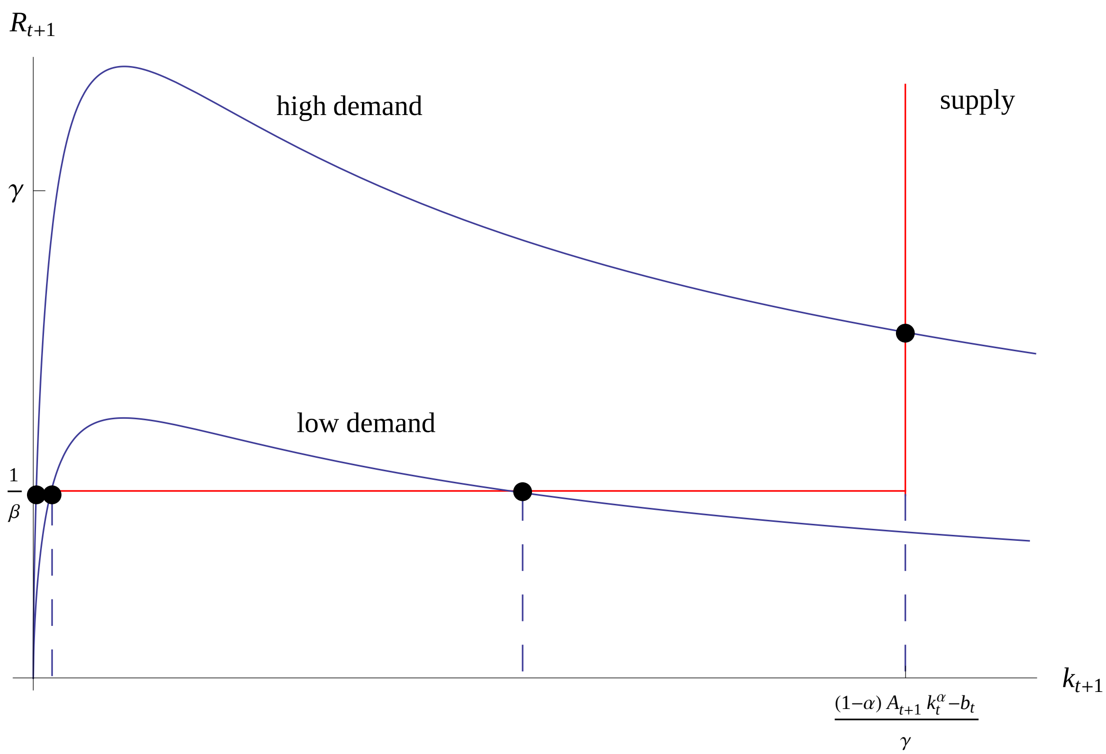


Figure 1: Capital market

Based on (9), the shape of the capital demand curve is interesting.[4] As depicted in

[4] In Figure 1, it is assumed that $\alpha A_{t+1} k_{t+1}^{\alpha-1} \geq \frac{\phi \alpha A_{t+1} k_{t+1}^{\alpha} + \lambda b_{t+1}^{N}}{k_{t+1} + \gamma^{-1}(1-\lambda) b_t}$. In general, the qualitative shape of the curve remains the same as in the figure, but it may not be differentiable at some points.

Figure 1, entrepreneurs may choose to have either low or high demand for capital for a given interest rate, and for a sufficiently high interest rate there is no demand at all. Strikingly, this is caused by the LTV ratio policy. Because entrepreneurs cannot fully use their bubbles as collateral, bubble holding has two contradicting effects on demand for capital. First, bubbles are partially used as bubbly collateral to relax the credit limit and raise capital investment. Second, the remaining part of the bubbles becomes a burden competing with capital investment over the loan. As a result, for any given interest rate, entrepreneurs may either choose to use most of the loan on bubble purchase and invest a little in capital or choose to heavily invest in capital to raise the fundamental collateral until the credit is sufficient for bubble purchase.

Figure 1 illustrates how varying the LTV ratio may affect capital market equilibrium. When the LTV ratio decreases, the capital demand curve shifts down, and vice versa. As a result, for a sufficiently high LTV ratio, capital is determined by capital supply, as shown at point E_h in the figure; whereas for a sufficiently low LTV ratio, capital is determined entirely by capital demand, as shown at point E_l in the figure.

In Martin and Ventura's (2015) original model, in which $\lambda = 1$, the capital demand is a standard downward-sloping curve. However, we find that for $\lambda < 1$, capital market equilibrium is not necessarily unique. Therefore, the selection of capital market equilibrium depends on coordination of belief of all entrepreneurs of each generation, increasing the economy's fluctuation in the short run. Across time, there may be infinite possible dynamical candidate paths of this economy, each of which may be chaotic. As a result, introducing a LTV ratio policy may lead to multiple equilibria, thus casting doubt on the effectiveness of the LTV ratio policy in dampening the credit cycle.

3 Steady-state analysis

Before analyzing steady-state properties in detail, we begin by characterizing the class of economy in which we are interested. Based on (10), it is necessary that $\bar{R} < \gamma$ at steady state. Because the minimum rate of interest is the rate of time preference, the first parameter characterization is as follows:

$$\gamma\beta > 1 \tag{11}$$

Next, it is helpful to analyze the bubbleless economy. If we let $\bar{b} = 0$, we can see that the LTV ratio completely vanishes. Therefore, our bubbleless economy is the same as that of Martin and Ventura (forthcoming). Here, we follow Martin and Ventura by considering only the dynamically efficient economy: By definition, the steady-state interest rate of the bubbleless economy without credit constraint is greater than the growth rate of the economy. In this model, this occurs when $\alpha > 0.5$. It is well known in rational bubble literature that, for bubbles to exist, the actual steady-state interest rate of the bubbleless economy must be below the growth rate of the economy. From (11), this means that the credit must be sufficiently constrained to suppress demand for capital and hence put the interest rate under the growth rate of the economy. This happens under the following parameter range:

$$0.5 < \alpha < \frac{1}{1+\phi} \tag{12}$$

According to (12), the higher the financial friction is, the more feasible bubbles can emerge.

We define two terminologies characterizing steady states as follows:

Definition 3.1.

1. The fully intermediated steady state is when the collateral is abundant and the capital stock is determined by fund supply.

2. The partially intermediated steady state is when the collateral is scarce and the capital is determined by fund demand. ■

Under (12), the parameter characterizations of the *bubbleless* economy being partially intermediated and fully intermediated are, respectively, given as follows:

$$0.5 < \alpha < \frac{1}{1+\gamma\beta\phi} \tag{13}$$

$$\frac{1}{1+\gamma\beta\phi} < \alpha < \frac{1}{1+\phi} \tag{14}$$

For the equilibrium dynamical system to have the steady state, the processes of technological progress and bubble creation must be convergent. For simplicity, we assume that $A_t = A$ and $b_t^N = b^N$. We denote $k_t = \bar{k}$, $b_t = \bar{b}$, and $R_{t+1} = \bar{R}$

as steady-state capital, bubbles, and interest rate, respectively. Satisfying (8)–(10), the steady state is as follows:

$$\bar{k} = \min\left\{\frac{1-\alpha}{\gamma}A\bar{k}^{\alpha} - \frac{\bar{b}}{\gamma}, \beta\left(\phi\alpha A\bar{k}^{\alpha} + \lambda b^{N} - \frac{1-\lambda}{\gamma}\bar{b}\right)\right\} \tag{15}$$

$$\bar{b} = \frac{\phi\alpha A\bar{k}^{\alpha} + \lambda b^{N}}{\gamma\bar{k} + (1-\lambda)\bar{b}}\bar{b} + b^{N} \tag{16}$$

In (15), the first argument of the right-hand side shows the fully intermediated steady state, and the second shows the partially intermediated one. If the steady state is fully intermediated, increase in bubbles competes with capital over savings, and hence bubbles crowd out investment. If the steady state is partially intermediated, increase in bubbles implies higher bubble creation, which helps to raise capital demand, and hence bubbles crowd in investment. However, the presence of a LTV ratio policy reduces this positive effect on capital demand because bubbles cannot be fully pledged and partially become a credit burden themselves.

In general, lowering the LTV ratio implies less benefit from bubble holding, and we would expect agents to hold fewer bubbles and invest more in capital. It is seemingly correct that agents would *tend* to hold fewer bubbles when the LTV ratio decreases.[5] However, how the LTV ratio influences steady-state capital depends on which type the steady state initially is. For the fully intermediated steady state, bubbles initially crowd out investment. Lowering the LTV ratio discourages bubble holding and hence raises capital. For the partially intermediated steady state, lowering the LTV ratio directly reduces demand for capital, but the size of bubbles remains unchanged because the interest rate is already at the lower bound, namely, the rate of time preference. The LTV ratio policy would be highly effective in the fully intermediated case, but it would be ineffective in the partially intermediated case.

The bubbleless economy is a good benchmark case to separate fully intermediated from partially intermediated steady states. If the bubbleless steady state is fully intermediated, the bubbly steady state must also be fully intermediated. If the bubbleless steady state is partially intermediated, tests of numerical examples

[5]This is not always true. Later in this section, we discuss the policy risk that arises from the existence of the other steady state: the low-capital steady state.

show that the bubbly steady state is usually also partially intermediated, except in some rare cases with very high new bubble creation.

More precisely, the steady-state system (15)–(16) has more than one root. Running various numerical examples, we find that there are at most two steady states.[6] The earlier analysis is what we found in the high-capital steady state, which is the same as that examined by Martin and Ventura in their 2015 paper. Below, we investigate whether the effectiveness of LTV ratio policy also holds true in the low-capital steady state.

Figure 2 shows how multiple steady states are determined by a numerical example of economy where the high-capital steady state is partially intermediated: $\beta = 0.9$, $\gamma = 1.5$, $A = 1$, $\alpha = 0.6$, $\phi = 0.4$, $\lambda = 0.8$, and $b^N = 0.001$.[7] Figure 2a shows what happens in the capital market, and Figure 2b shows how savings are correspondingly divided.

As discussed earlier, the high-capital steady state (E_2) is the partially intermediated steady state. In this case, Figure 2b indicates that only savings (solid blue curve) are not fully used on capital and bubbles, and are instead partially consumed. However, there is also the low-capital steady state (E_1), which is fully intermediated and contains larger bubbles.

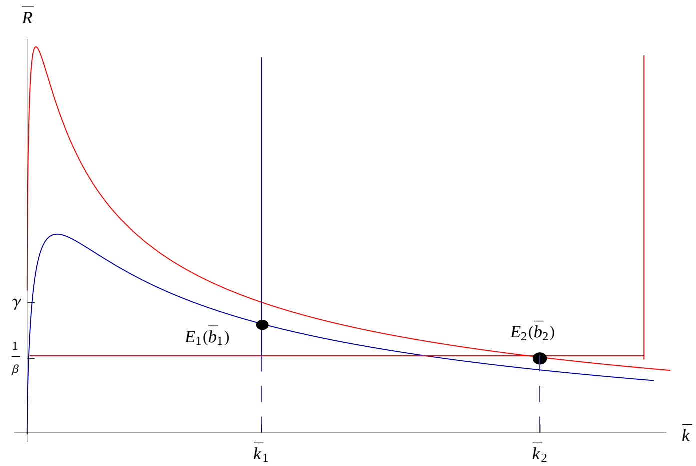
Such multiplicity is a concern regarding policy implication: We cannot know which steady states would prevail and hence which effects we should expect from the LTV ratio policy. Theoretically, one way to eliminate this multiplicity is to check the stability of each steady state. Basically, we can ignore all unstable steady states because any small perturbation would bring the economy toward another steady state.

Remark 3.1.

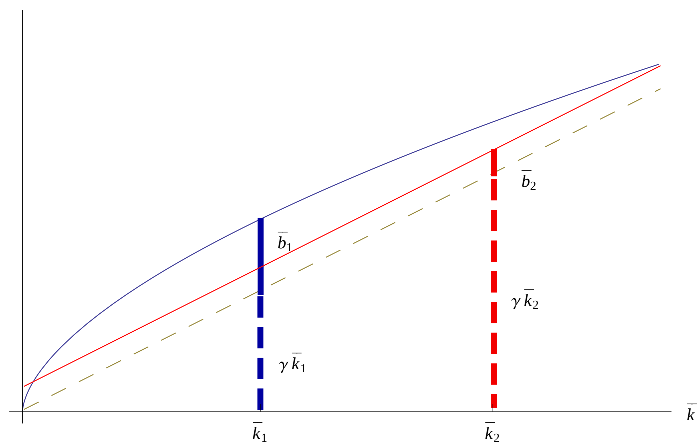
1. The partially intermediated steady state is always a sink where the local

[6] Within the parameter range concerned, most of the numerical examples give two steady states, some have a unique steady state, and the remainder have no consistent real roots. Moreover, the multiplicity of steady states does not result from LTV ratio policy, because the same result holds in Martin and Ventura's (2015) original model.

[7] In the literature, the standard value of discount rate and capital-income share are 0.999 and 0.633, respectively, which are close to our example. Given that one period in our model is roughly 30 years, $\gamma = 1.5$ implies the annual growth rate is about 1.4 percent, which is reasonable. We choose high financial friction $\phi = 0.4$ to satisfy (13).



(a) Capital market



(b) Division of savings

Figure 2: Multiple steady states

dynamic processes of capital are as follows:

$$\left(k_{t+1} - \bar{k}\right) = \frac{1}{\gamma\beta}\left(k_t - \bar{k}\right)$$
$$\left(b_{t+1} - \bar{b}\right) = \frac{1}{\gamma\beta}(b_t - \bar{b}$$

2. The fully intermediated steady state can be stable, saddle-stable, or unstable

depending on its two eigenvalues (ν_1, ν_2) as follows:[8]

$$\nu_1 = \frac{z_1 + z_4 - \sqrt{(z_1+z_4)^2 - 4(z_1 z_4 - z_2 z_3)}}{2}$$
$$\nu_2 = \frac{z_1 + z_4 + \sqrt{(z_1+z_4)^2 - 4(z_1 z_4 - z_2 z_3)}}{2}$$

where

$$\begin{aligned}
z_1 &= \frac{\alpha(1-\alpha)A}{\gamma \bar{k}^{1-\alpha}} \\
z_2 &= \frac{1}{\gamma} \\
z_3 &= z_1 \left[\frac{\left(\gamma\bar{k} + (1-\lambda)\bar{b}\right)\left(\frac{\phi\alpha^2 A\bar{b}}{\bar{k}^{1-\alpha}}\right) - \gamma\bar{b}\left(\phi\alpha A\bar{k}^{\alpha} + \lambda b^N\right)}{\left(\gamma\bar{k} + (1-\lambda)\bar{b}\right)^2} \right] \\
z_4 &= \frac{\left(\gamma\bar{k} - \bar{b}\right)\left(\phi\alpha A\bar{k}^{\alpha} + \lambda b^N\right) - \left(\gamma\bar{k} + (1-\lambda)\bar{b}\right)\left(\frac{\phi\alpha^2 A\bar{b}}{\gamma\bar{k}^{1-\alpha}}\right)}{\left(\gamma\bar{k} + (1-\lambda)\bar{b}\right)^2}. \quad \blacksquare
\end{aligned}$$

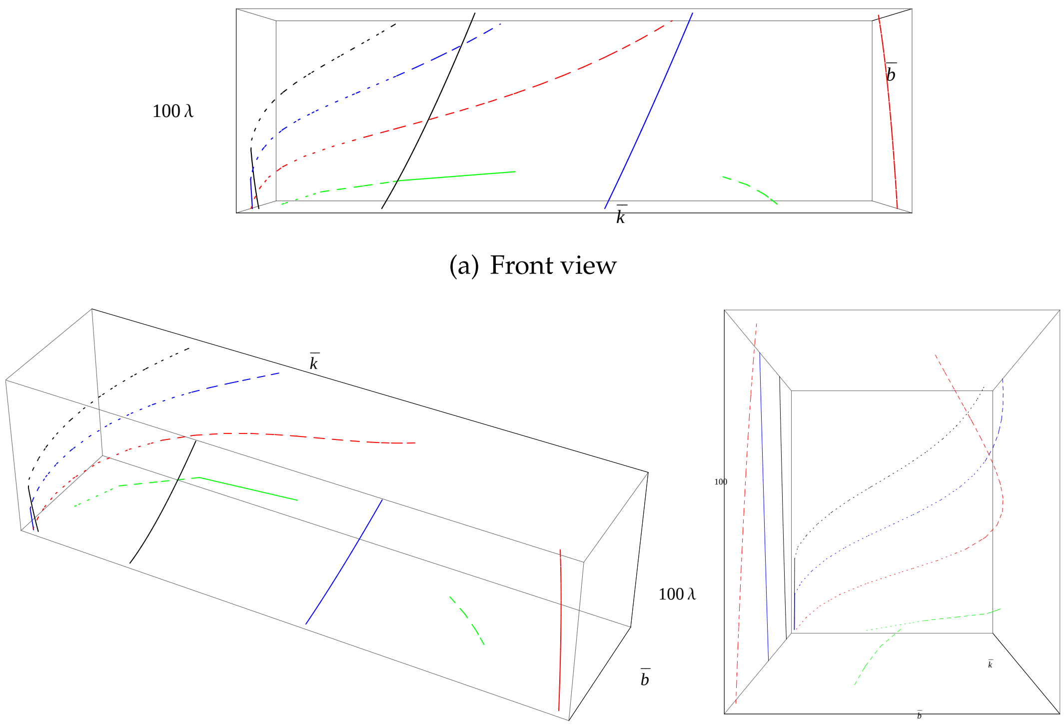
From Remark 3.1 and (11), all partially intermediated steady states are sinks, which means that their nearby points would monotonically approach them. For fully intermediated steady states, their stability is ambiguous and determined case by case. Unfortunately, most of the low steady states are fully intermediated.

Figure 3 shows the results of numerical comparative statics of steady-state capital and bubbles by varying financial friction (ϕ) and the LTV ratio (λ): For this numerical example, $\beta = 0.9$, $\gamma = 1.5$, $A = 1$, $\alpha = 0.6$, and $b^N = 0.0005$. Black, blue, red, and green in the figure represent ϕ equal to 0.3, 0.4, 0.5, and 0.6, respectively. The dotted, dashed, and solid lines denote the steady state as unstable, saddle-stable,[9] and stable, respectively.[10] Unfortunately, although we had hoped to rule out the low-capital steady state because of its unstable property, this did not occur. Figure 3 demonstrates that there are many cases where the low-capital steady state is either stable or saddle-stable.

[8] These results of stability can be trivially proved by using a standard linearization method, and hence the proof is not shown in this paper.

[9] The saddle-stable steady state means that there is a local saddle path where any point on the path would dynamically move along this path toward the steady state. As a result, for any given capital in the neighborhood, there is a particular level of bubbles that can be in equilibrium.

[10] The far-right red line in Figure 3a-b is a dashed line as can be seen in 3c. However, because of the illusion of perspective, it appears as a solid line.



(a) Front view

(b) Overall view

(c) Side view

Figure 3: Multiple steady states and their stabilities

Therefore, we also need to consider the effects of LTV ratio policy on the low steady state. From various numerical investigations, observations are fairly consistent, as shown in Figure 3. From 3a, we can observe that a full general pattern seems to be that as the LTV ratio is decreasing, the low-capital steady-state capital is also decreasing, but finally reverses at the very low level of LTV ratio.[11] The reversal occurs when the steady state changes from fully intermediated to partially intermediated. As can be seen from the low-LTV-ratio capital demand curve in Figure 1, if we set the LTV ratio even lower, the curve would shift down and the low-capital steady-state capital would indeed increase. This is because at the low level of capital the marginal product of capital is so high that investing more in capital significantly increases fundamental collateral and raises capital demand.

In terms of the effects of LTV ratio on low-capital steady-state bubbles, testing various numerical examples shows a general pattern similar to that in Figure 3c: as the LTV ratio decreases, bubbles grow, then reverse, continue to decrease, and

[11]One may think that the red and green low-capital steady-state lines hit the non-negativity constraint of the LTV ratio before they reverse.

finally stay constant at the lowest partially intermediated steady-state.[12] Surprisingly, there are cases where decrease in the LTV ratio actually leads to increase in bubbles. This happens at the capital level that is not too low and not too high. At this range of capital in the low steady state, agents may choose to excessively hold bubbles. Given such a scenario, reducing the LTV ratio may instead encourage more bubble holding, which in turn causes lower investment in capital. Consequently, with such complicated and somewhat ambiguous patterns, implementing a LTV ratio policy carries a risk: The intention to reduce fluctuation driven by bubbles leads to an increase in bubbles.

So far, we have discussed bubbles as a source of fluctuation without carefully describing how the economy may fluctuate because of them. The next two sections formally specify these fluctuation issues by means of sunspot equilibria and how LTV ratio policy affects them.

4 Sunspot equilibria analysis

Figure 3 shows that for given ϕ and λ, there are two steady states. If both of them are stable or saddle-stable, it is theoretically well known that sunspot equilibria can be constructed where there is a probability for the equilibrium to switch from one steady state to the other.[13] Notably, such sunspot equilibria generate fluctuation between high-capital and low-capital steady states for a given value of b^n: the parameter of bubble creation.

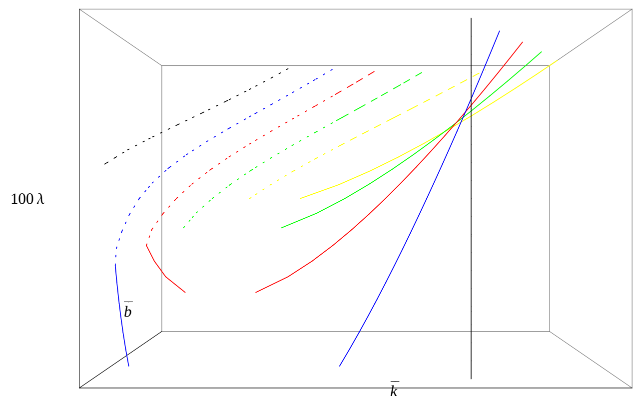
Recall that bubble creation (b^N) is the equilibrium-consistent belief that agents take as given before making an economic decision. In particular, it is the fluctuation that occurs despite no change in belief of future bubble creation. It represents how much agents believe that new bubbles would appear in the future, which we interpret here as market sentiment: Higher bubble creation implies higher market sentiment. With different market sentiment, the steady state of economy differs. Thereby, the fluctuation caused by sunspot equilibria occurs even without any change in market sentiment. However, this implies another type of fluctuation

[12] When the steady state is partially intermediated, (16) reduces to $\bar{b} = \frac{\gamma\beta}{\gamma\beta - 1} b^N$.

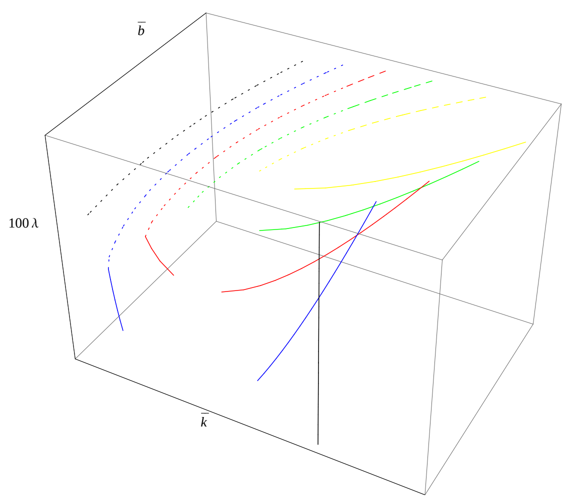
[13] Formally, we assume the two-state Markov process where switching probabilities from low-capital to high-capital and high-capital to low-capital steady states are close to zero, so that none of our model formalization changes.

caused by the sunspot equilibria over steady states of economies with different market sentiments.

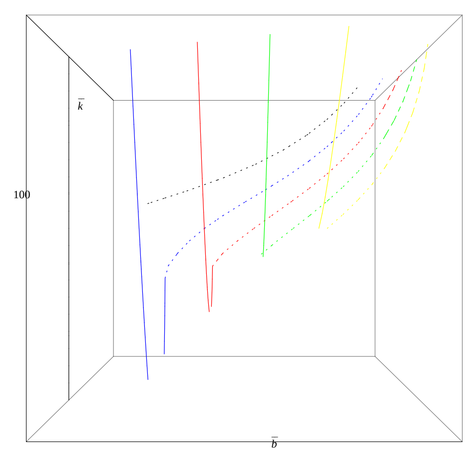
Figure 4 shows the comparative statics of steady states by varying market sentiment and the LTV ratio: In this numerical example, $\beta = 0.9$, $\gamma = 1.5$, $A = 1$, $\alpha = 0.6$, and $\phi = 0.3$. Black, blue, red, green, and yellow represent b^N equal to 0, 0.0005, 0.001, 0.0015, and 0.002, respectively.



(a) Front view



(b) Overall view



(c) Side view

Figure 4: Multiple steady states of different market sentiments

Because market sentiments are simply consistent beliefs, switching among beliefs are plausible. We can construct the sunspot equilibria where the economy can switch back and forth between a steady state of low market sentiment to a steady state of high market sentiment.

Figure 5 visually summarizes two sources that sunspot equilibria can be built upon and hence create fluctuations. The solid blue arrows in the figure represent sunspot equilibria switching between different steady states (low capital and high capital) for a fixed market sentiment. The dashed red arrows represent sunspot equilibria switching between, for example, high-capital steady states from different market sentiments. Differentiating the two sources of fluctuation is useful in understanding the effects of LTV ratio policy on each of them, which is the task of the next section.

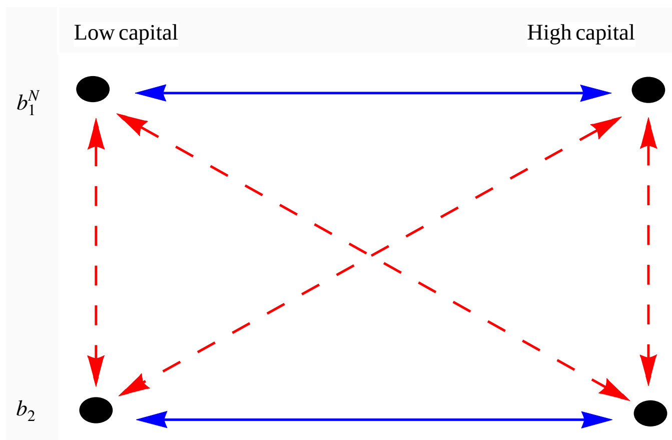


Figure 5: Two sources of fluctuations: multiplicity of steady states and market sentiments

5 Stabilization policy: LTV ratio versus fluctuation

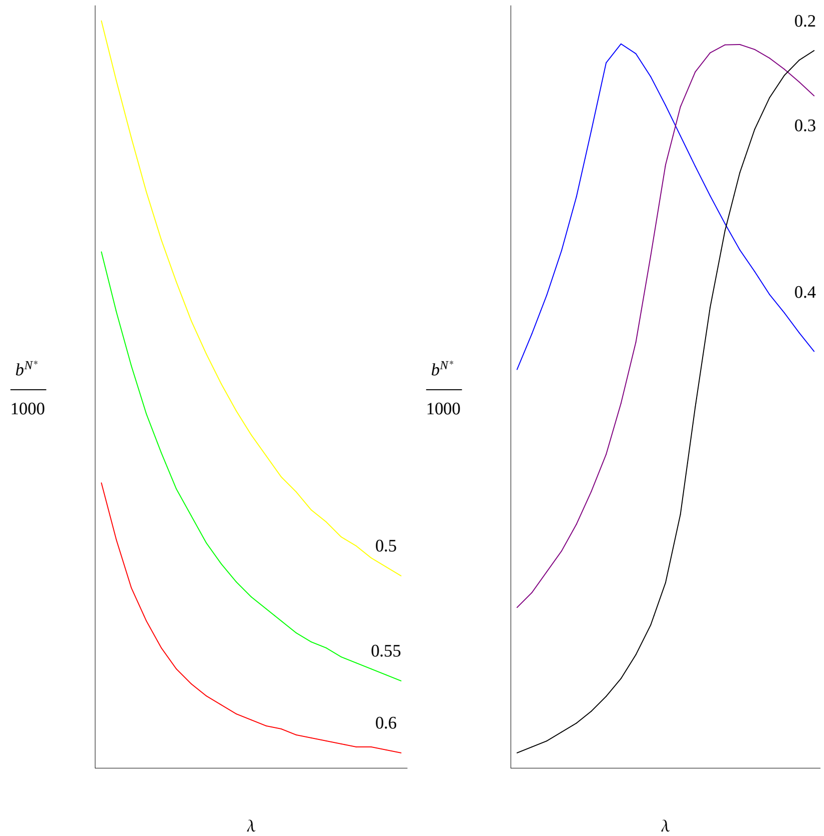
First, we consider how the LTV ratio may help to reduce fluctuation caused by sunspot equilibria within the same market sentiment. In fact, Figure 3 already shows this issue. By lowering the LTV ratio, the low-capital steady state starts to lose its stability. When the low-capital steady state is unstable, the sunspot equilibrium cannot be constructed because it would not be a rational expectations equilibrium. Hence, the fluctuation is eliminated. However, at a sufficiently low LTV ratio, the low-capital steady state may reverse to become stable again. Various numerical examples confirm this result, and this reversal occurs when the low-capital steady state changes from fully intermediated to partially intermediated. Remark 5.1 summarizes this result.

Remark 5.1. For a given market sentiment, a decrease in the LTV ratio can reduce the fluctuation by destabilizing the fully intermediated low-capital steady state and returns the economy to a unique steady state. ■

Next we consider how the LTV ratio may affect economic fluctuation caused by change in market sentiments. Notably, different market sentiments result in different steady states. However, this does not mean that any type of belief would be consistent: There may be no supporting real-value steady state. From the various numerical investigations, we find that $[0, b^{N*}]$ is a feasible range of market sentiment, where b^{N*} is the maximum market sentiment for a given economy. Intuitively from (16), the higher market sentiment raise steady-state bubbles, but bubbles cannot be arbitrarily large, because of limited available savings. Interestingly, we find a connection between LTV ratio policy and b^{N*}.

Figure 6 shows how the LTV ratio influences maximum market sentiment in the economy with different degrees of financial friction: This comparative statics study is conducted using the earlier example where $\beta = 0.9$, $\gamma = 1.5$, $A = 1$, $\alpha = 0.6$, and $\phi = 0.3$. The number labeling each curve is the degree of financial friction (ϕ). Again through various numerical examples, we find that if the bubbleless economy is fully intermediated, a decrease in the LTV ratio always raises the maximum market sentiment, as shown in Figure 6a. Conversely, if the bubbleless economy is partially intermediated, there tends to be a particular LTV ratio that maximizes the market sentiment, and this maximizing LTV ratio approaches 1 when the financial market friction becomes more severe, as shown in Figure 6b. In general, as the financial friction becomes more severe, the maximizing LTV ratio continuously changes from left-corner-solution 0, through interior-solution $(0, 1)$, to right-corner-solution 1.

Having less maximum market sentiment means that there are fewer feasible beliefs that agents could take and hence fewer sunspot equilibria that could be constructed. In this respect, we can reduce the fluctuation. This is the strikingly new result in the theoretical perspective because now the objective function is to minimize the range of consistent beliefs, which can only be done in the bubble-creation framework. Unfortunately, studying this issue analytically appears to be impossible.



(a) Fully intermediated bubbleless economy

(b) Partially intermediated bubbleless economy

Figure 6: LTV ratio effects on maximum market sentiment

Remark 5.2.

1. If financial friction is sufficiently low that the bubbleless economy is fully intermediated, lowering the LTV ratio expands the feasible set of market sentiment and hence increases the fluctuation from different market sentiments.

2. If financial friction is sufficiently severe, which implies that the bubbleless economy is partially intermediated, lowering the LTV ratio reduces the feasible set of market sentiments and hence decreases the fluctuation from different market sentiments. ■

From our earlier discussion, Remark 5.2 has some policy implications. In particular, it highlights the effectiveness of LTV ratio policy in reducing the fluctuation caused by changes in market sentiments. A possible explanation would be that severe financial friction greatly limits fundamental collateral, and hence bubbly collateral is relatively more important. By restricting bubbly collateral, agents lose confidence, and high market sentiment can no longer be achieved. This is beneficial because it is empirically known that emerging-market economies normally suffer from financial friction, and hence a LTV ratio policy can help to stabilize these economies, at least regarding across-market-sentiment fluctuation. Conversely, a LTV ratio policy may be ineffective in reducing such fluctuation in developed countries where financial friction is relatively low. Intuitively, low financial friction already induces high demand for capital and hence a high interest rate. High market sentiment may push the interest rate above the growth rate of the economy, which would violate the bubble condition. Consequently, lowering the LTV ratio creates more room for high market sentiment to have supporting rational expectations equilibrium.

6 Conclusion

We modified the bubble-creation model of Martin and Ventura (forthcoming) by incorporating a particular macroprudential policy, namely LTV ratio. According to the literature of rational bubbles, bubbles help to relax the credit limit and hence can raise investment in capital. Implementing a LTV ratio policy is expected to discourage bubble holding, which is considered a source of economic fluctuation, and unfortunately, to lower capital investment.

This logic is partially correct and incorrect. In the short run, the LTV ratio in fact also causes multiplicity, which may result in chaotic equilibrium dynamics. In the long run, the logic is partially correct in the high-capital steady state, but it fails to explain the low-capital steady state, which has more complicated behavior. This ambiguous behavior of the low-capital steady state makes a LTV ratio policy naturally risky. In particular, lowering the LTV ratio may instead raise bubbles while investment still decreases.

Moreover, the LTV ratio itself can affect fluctuation. On the one hand, lowering the LTV ratio may destabilize the fully intermediated low-capital steady state, and therefore the fluctuation caused by multiplicity of steady states is eliminated.

On the other hand, lowering the LTV ratio may reduce the range of feasible market sentiments, or set of consistent bubble-creation beliefs, which would lead to reduction of fluctuation caused by changes in market sentiments.

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